STUDIES ON LATERALLY LOADED PILES

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by

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CERTIFICATE

Certified that the work presented in this thesis has been carried out by Sri K. Madhavan under our supervision and has not been submitted elsewhere for a degree.

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DEDICATION

To M.R. Madhav, my ideal teacher

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NOTATIONS

A_y - Non-dimensional term representing deflection.

B - Width of the pile.

EI - Modulus of rigidity of the pile.

k - Coefficient of subgrade reaction.

K - Maximum value of k at the lower end of the pile.

L - Length of the pile.

M - Moment

m - Ratio of stress in the soil at the top to that at the bottom of the pile as used in chapter v.

m - Stiffness factor as used in chapter iv.

m_o - Mass ratio.

m_1 - Mass per unit length of the pile.

m_2 - Mass of soil that takes part in the vibrations along with the pile.

V - Shear force

m_h - Ratio between coefficient of horizontal subgrade reaction and depth below surface.

n - Any real positive quantity greater or less than unity.

n_h - Constant of horizontal subgrade reaction.

P - Static lateral load.

P(t) - Dynamic lateral load.

p - Pressure per unit length.

q - Yield stress of the soil.

R - Load ratio.

T - Relative stiffness factor.

T_o - Time factor.

X - Distance along the length of pile.

X_1 - Depth of the point at which the soil behaviour changes from plastic to elastic.

y - Deflection of the pile.

$y\ 1$ - Deflection of the top of the pile in the elasto-plastic case.

$y\ 2$ - Deflection of the bottom of the pile in the elasto-plastic case.

Z - Non-dimensional factor.

Z_{max} - Maximum value of Z.

r_o - Stiffness ratio as used in chapter III.

r - Stiffness ratio as used in chapter IV.

α - The ratio of Z_1 to Z_{max}, Z_1 being the depth at which the soil behaviour changes from plastic to elastic as used in chapter IV.

μ - Poisson's ratio of the soil.

σ - Stress.

ϵ - Strain.

ω - Frequency of the forced vibration.

CHAPTER I

INTRODUCTION

1.1 GENERAL:

The purpose of piles is to transfer the loads from a footing to a soil that can carry such loads with settlements not exceeding the permissible values. Piles have been used even from very early times to support structures. Timber piles were used since they were abundant and the labour was also cheap. As many number of piles as the ground would take were driven since the behaviour of the piles was not known. Later with the advances in various fields, new materials, and new methods of design have been arrived at.

The piles were mainly designed to carry vertical loads that come from the superstructure. The analysis of foundation for carrying vertical loads follows a conventional procedure but the analysis for lateral loads poses a complex problem. When the lateral force coming on the vertical piles is of smaller magnitude, no provision is made and when it exceeds the allowable amount batter piles are generally resorted to (40). The modern codes specify the minimum lateral capacity required of the piles.

The California Building Code specifies it as 20% of the axial load (42). The allowable horizontal load on vertical piles a given by Mcnulty (20) is shown in table I. Generally batter piles are provided when the lateral load exceeds 1000 lb/pile (40). But the driving cost is prohibitively high.

1.2 LATERAL LOADS:

Later loads can be grouped under three headings:

(a) Short term static loads

(b) Long term static loads

(c) Dynamic loads.

The lateral loads can be caused by the lateral earth pressure, wind forces, water forces, ship impact, traction forces from automobiles or trains. So when the piles are used as supports for retaining walls, bridge abutments, p'ers fenders, dolphins, anchors for bulkheads and waterfront and off-shore structures.

The literature pertinent to the problems analyzed has been reviewed in chapter II. Specifically various studies on coefficient of subgrade reaction have been critically examined.

1.3 DYNAMIC LATERAL LOADS.

These may be caused by earthquakes, wave action

and also by the water current pressure. When piles are used as supports to some of the machine foundations, they may be subjected to horizontal dynamic loads.

The dynamic response of a pile loaded laterally at the top by a harmonic loading is analyzed in chapter III. The masses of the pile and a certain amount of soil participating in the vibrations, are included in the solution.

1.4 SOIL - PILE INTERACTION:

Both the flexural and axial stresses have to be considered for the design of the piles. The bending stresses being the major factor, has to be predicted to the correct values. So the interaction between the soil and the pile is to be carefully analysed when the lateral load is acting. Since this problem deals with the soil as well as the pile, a knowledge of the behaviour of both is necessary.

The soil characteristics are to be found first and later the behaviour of the pile in action with the soil is to be studied to find the flexural stresses in the pile. The soil is not perfectly elastic at all strains. At small strains it behaves elastically and at large strains it behaves inelastically and the problem is further complicated to analyse

when the soil behaves inelastically. So in the vicinity of the pile, when it is strained to a larger extent it is in the inelastic range and it is more so at the top of the pile.

Also the deflections are linear with the load upto ½ to 1/3 of the ultimate lateral load. At higher load levels nonlinearity holds good and when the deflection is about 20% of the lateral dimension of the pile the nonlinearity comes into the picture (5).

Chapter IV discusses the pile response due to lateral loads in a soil that is characterized by elasto-plastic behaviour. This problem has been analyzed and discussed in detail. Closed form solutions are given for piles that are hinged or fixed at the bottom.

Lastly, in chapter V expressions are derived for vertical stresses in an elastic soil due to the action of a short (rigid) pile. The thesis ends with a list of references.

TABLE I

ALLOWABLE HORIZONTAL LOAD ON VERTICAL PILES
(for ¼" lateral movement)

Type of pile	Pile head	Type of soil	Allowable load lb/pile
Timber	Free end	Sand	1500
		Medium clay	1500
	Fixed end	Sand	4500
		Medium clay	4000
Concrete (16"ø)	Free end	Medium sand	7000
	or fixed end	Fine sand	5500
		Medium clay	5000

CHAPTER II

LITERATURE REVIEW

2.1 STATIC SOIL MODULUS:

The behaviour of the pile mainly depends upon the interaction between the soil and the pile as explained. So before determining the actual behaviour of the pile it is necessary to know the soil properties that affect the behaviour. One such property is the soil modulus.

Modulus of subgrade reaction is usually defined as the pressure required per unit deformation calculated at 0.05" of deformation. Terzaghi named it as the "coefficient of subgrade reaction". According to Terzaghi (35) it indicates the pressure per unit area of the surface of contact between a loaded beam or slab and the subgrade on which it rests and on to which it transfers the loads. In other-words it is the ratio between the pressure at any given point of the surface of contact and the settlement produced by the load application at that point.

$$k = \frac{p}{y} \quad \ldots\ldots\ldots\ldots \quad (2.1)$$

where, k - coefficient of subgrade reaction or soil modulus.

p - pressure per unit length.

y - deflection.

In the analysis of laterally loaded piles, k is usually express as a function of the depth of the pile.

$$k = K \left(\frac{x}{L}\right)^n \quad \ldots\ldots\ldots\ldots \quad (2.2)$$

where, x - depth along the pile

L - length of the pile

For stiff clays K is constant and for sands it is linearly varying. So in the Eqn. (2.2), the value of 'n' is zero for clays and is equal to one for sands. For any other type of soil, a suitable value of n can be chosen between zero and one. Fig. (2.1) shows the variation of k with depth for different types of soil (43).

For stiff clay the coefficient of horizontal subgrade reaction has the same value as that of the vertical subgrade reaction. For cohesionless subgrades the value of the coefficient of horizontal subgrade reaction is determined by

$$k = m_h \, x \quad \ldots\ldots\ldots\ldots \quad (2.3)$$

where, 'm_h' is assumed to be the same for every point of the surface of contact (35).

Broms (7) has defined it as

$$k = n_h \, \frac{x}{B} \quad \ldots\ldots\ldots\ldots \quad (2.4)$$

where, 'n_h' is a constant which varies with the

relative density of the soil and the location of the water table. 'B' is the width of the pile. So it is obvious that k increases with depth and decreases linearly with the width.

In normally consolidated clays k varies linearly with depth and in the over consolidated clays k remains constant (5,13) Reeves (46) indicated that the average of a linearly varying soil modulus should be 1/3 the modulus at the bottom of the pile.

According to Kubo's study (19), the value of k remains practically constant for a width greater than 20 cm. The soil modulus k varied widely not only with depth but also with deflection (12). For varying loads k is not constant at any given depth.

When the lateral load is increased, the reaction of the soil first increases, reaches a maximum, and then decreases. Simultaneously the deflection decreases. So k ultimately decreases as the load increases.

Chang (8) assumed the soil as an elastic medium and gave the relationship as per Eqn. (2.1).

Hayashi et al., (15) carried out many field tests on full scale prototype piles and investigated the characteristics of a horizontal surface reaction.

As a conclusion they obtained the relationship between p and y as

$$p = k \quad B \ y^{0.5} \qquad \ldots\ldots\ldots \quad (2.5)$$

Glesser (2) predicted that "it is quite possible that soil modulus can vary as some power of the depth or some function of the deflection. Rifaat (36) also conceived the idea that the soil modulus is a function of the depth along the pile. Siva Readdy and Valsangkar (33) have assumed the k to be of the form

$$k = k_o + k_1 \ x^2 \qquad \ldots\ldots\ldots (2.6)$$

Shinokara (47) and Kubo (48) showed that the results of many systematic model tests lead to the relationship

$$p = k \ B \ x \ y^{0.5} \qquad \ldots\ldots\ldots (2.7)$$

Ripperger in his discussion (12) defines it as the secant of one of the load deflection curves. Also he adds that for a perfectly elastic soil the modulus for a given pile and a given soil would be single valued but for an inelastic soil there would be an infinite number of modulii. The definition in the case of an inelastic soil is made some what more significant by considering the modulus at an arbitrarily chosen deflection.

No unique value can be attributed for a given soil. The pile deflection varies with the pile

size, the pile stiffness, the magnitude of the load, and the manner of load application, the elastic properties of the soil and hence the modulus will also vary with these factors. Therefore the soil modulus exists only as a mathematically convenient expression for the ratio of the soil reaction to pile deflection.

2.2 DYNAMIC SOIL MODULUS·

Repetitive loads decrease the soil modulus (5). Gaul (13) also has assumed a constant soil modulus in the case of the dynamically laterally loaded pile for the soil used i.e. the montmorillonite rich bentonite and concluded from the experiments that the dynamic soil modulus differs from the static one by less than 0.1%.

Finally it may be said that it is unnecessary to determine the soil modulus very accurately because a large variation in it will produce only a small difference in the value of β where

$$\beta = \sqrt[4]{k/4\,E\,I} \qquad \ldots\ldots\ (2.8)$$

E - young's modulus of the pile

I - moment of Inertia of the pile.

2.3 STATIC LATERAL LOAD

The settlement of piles under vertical loads during driving was observed for the first time by

Personet in eighteenth century. The load displacement of the piles loaded laterally was observed for the first time by Sandeman in 1880.

Feagin (41) conducted some lateral load tests on single and on group of piles of timber and concrete to determine their resistance. He concluded that for less than 6.5 tons of load per pile the group effect is not felt on the deflection and for higher loads the deflection appears to be less for the smaller group and also that the stability increases in the case when the pile is carrying vertical loads. The deflection was found to increase in the case of repetitive loading than in the case of the sustained load.

Chang (8) has given the analytical solution for the static lateral loading. Assuming the pile analogous to the beam on elastic foundation he solved the equation

$$E\,I\ \frac{d^4y}{dx^4} = p \qquad \ldots\ldots\ldots (2.9)$$

and gave the solution as

$$y = C\,e^{-\beta x}\,(\cos\beta x + \sin\beta x) \quad \ldots \quad (2.10)$$

and the constant is to be found by using the boundary conditions.

Cummings (8) assumed a power series for the deflected pile and for the soil modulus increasing linearly with the depth of the pile. He determined the relationship between the horizontal load and the top deflection of the pile.

Maccamon et al., (2) tested the hollow piles loaded with static lateral loads at the top. He concluded from the results that the soil acted as an elastic medium when resisting the lateral forces. Also that the length of the pile does not influence the point of occurrence of the maximum bending moment. From the strain gage readings, the bending moment was obtained.

Feagin (2) conducted load tests on groups of battered and vertical piles. He concluded

(a) that the groups of battered piles combined with vertical piles are more resistant to lateral loads either against or in the direction of the batter than are corresponding groups of vertical piles.

(b) that the resistance to a lateral load against the batter is greater under a vertical load, whereas for a lateral load in the direction of the batter, the resistance is substantially the same either with or without vertical load.

(c) that the resistance to latteral loads in the direction of the batter generally exceeds that for

loads against the batter for similar pile group arrangements either with or without a vertical load and

(d) that a group of piles battered in both directions is more resistant to lateral loads than a group battered in one direction.

Based on the results of model tests Tschebotarioff (2) concluded that the resistance per pile to lateral loads decreases appreciably with an increasing number of piles in a group.

Wagner (2) conducted tests on individual timber piles and concluded that

(a) The overdriving reduces the lateral resistance of a pile.

(b) Increasing the length of a pile does not improve its lateral resistance, provided the pile is embeded sufficiently to prevent movement in the lower portion.

(c) Increasing the size of the pile increases lateral resistance.

Glesser (2) and Palmer and Brown (3) analytically solved the problem by the use of the "difference equations". Reese and Matlock (21) has given the generalised solutions for the laterally loaded piles assuming k increases with depth because of the increase in soil strength. Also they have indicated that the k can be parabolic.

Siva Reddy and Valsangkar (33) have given the generalised solutions of a laterally loaded piles with polynomial variation of soil modulus. They solved the differencial equation by the series method.

Davisson and Prakash and Broms (5,6,7) concluded that the pile can no longer be approximated as a semiinfinitely long beam when the length βL is less than about two.

Kubo (19) carried out tests on model steel piles embeded in sand and came out with the results that

(a) Soil reaction $p = k \times y^{0.5}$

(b) The effective length of the pile is not more than 1.5 l where l is the depth of the first zero point of moment curve. Also he has classified the soil into two types.

(i) S - type soil when p - y relationship is governed by the relation $p = k \times y^{0.5}$ example sand layer of uniform density, normally consolidated clay layer.

(ii) C - type soil where p - y relationship is governed by $p = k\ y^{0.5}$ example highly precompressed clay layer.

2.4 DYNAMIC LATERAL LOADS :

Hayashi (15) with the help of the model

pendulum, studied the dynamic response of piles subject to harmonic excitation. He has only estimated the vibration characteristics of the pile such as the relation between the frequency and deflection for both forced and free vibrations.

Gaul (13) performed tests in a dimensionally scaled model of a vertical pile under static and dynamic loads. With the help of a mechanical oscillator, the dynamic load was applied. Electric strain gauges were attached to the pile. The strain variations were amplified on an oscilloscope.

Based on the analysis of the data obtained he conclued that

(a) the pile vibrates in the form of a standing-wave which is in phase with the oscillating load.

(b) Damping of soil was negligible.

(c) At relatively low frequency of load oscillations, the maximum bending moment and the section where it occurs are not materially different as compared with a static case.

(d) The soil modulus is constant for the soil used under dynamic load.

Hayashi and Miyajima (49) performed tests on vertical steel H piles embeded in sand and subjected

to static and dynamic loads. They have conducted both the free and the forced vibration tests, and measured the bending moment, the displacement and acceleration of the pile head. They have concluded that

(a) Natural frequencies and response curves of single vertical piles could be calculated by conceiving a simplified system of vibration and the results of calculations agree with those obtained by the free and forced vibration tests.

(b) Damping coefficient measured in the free vibration tests depended on the relative density of the subgrade and on the length of the free parts of the pile.

Prakash and Aggarwal (27) conducted dynamic tests on Aluminum model piles embedded in sand and using the vibration table . A proving frame was designed to measure the frequency of vibration. His findings were

(a) Under steady state dynamic loading the soil around the pile gets compacted initially to a large extent till optimum compaction is reached.

(b) The initial static strength of a pile is less than the steady state dynamic strength. But after the soil gets compacted to a large degree

the static strength is larger than the dynamic strength.

(c) The zone of influence of a dynamically loaded pile extends to a considerably great distance than that for a statically loaded pile.

(d) The damping of the soil-pile system is about 30%.

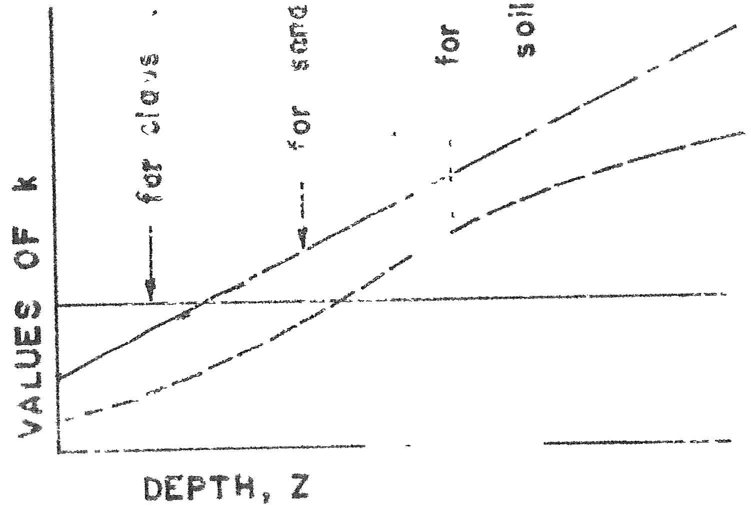


FIG (2 I) VARIATION OF k WITH DEPTH FOR ... T TYPES OF SOILS

CHAPTER III

DYNAMIC RESPONSE OF LATERALLY LOADED PILE

3.1 INTRODUCTION:

Experimental work has been done on dynamically laterally loaded piles by Gaul (13) and Hayashi et al., (15). Analytical solutions have not been attempted os far. Here an analytical solution is presented.

3.2 FORMULATION OF THE PROBLEM:

Fig.(3.1) shows the pile diagramatically in which

m = mass that is taking part in the vibrations

$= m_1 + m_2$

where, m_1 = mass per unit length of the pile.

m_2 = mass of soil that takes part along the unit length of the pile.

V = Shear force

M = Moment

dM, dV = small increments in bending moment, shear force respectively.

$P(t)$ = dynamic load.

ω = frequency of the forced vibration.

ASSUMPTIONS:

1. Damping is negligible

2. The soil behaves like a Winkler foundation.

Considering an element dx of the pile, the forces acting are shown in Fig. (3.1). Writing down the equations of equilibrium,

$$\Sigma H = 0 \; ; \; \frac{\partial V}{\partial x} = - p + m \ddot{y} + k y \quad . . . . (3.1)$$

$$\Sigma M = 0 \; ; \; \frac{\partial M}{\partial x} = V \quad . . . . (3.2)$$

But $M = - E I \dfrac{\partial^2 y}{\partial x^2}$

$$\frac{\partial^2 M}{\partial x^2} = - E I \frac{\partial^4 y}{\partial x^4} \quad . . . . (3.3)$$

Combining Equns. (3.1),(3.2) and (3.3),

$$E I \frac{\partial^4 y}{\partial x^4} + m \frac{\partial^2 y}{\partial t^2} + k y - p = 0 \quad . . . (3.4)$$

The pile is not loaded along the length, i.e., p = 0 and Equn. (3.4) reduces to

$$\frac{\partial^4 y}{\partial x^4} + \frac{m}{EI} \frac{\partial^2 y}{\partial t^2} + \frac{k}{EI} y = 0 \quad . . . . .(3.5)$$

Equn. (3.5) is to be solved using the following boundary and initial conditions.

CASE I: <u>TOP FREE AND BOTTOM HINGED.</u>

$$t = 0, \; x = 0; \quad E I \frac{\partial^2 y}{\partial x^2} = 0$$

$$- E I \frac{\partial^3 y}{\partial x^3} = - P(t) = -P e^{i\omega t}$$

$$. . . (3.6)$$

$$x = L \qquad E I \frac{\partial^3 y}{\partial x^3} = 0$$

$$E\,I\,\frac{\partial^2 y}{\partial x^2} = 0$$

CASE 2: $t = 0, \quad x = 0 \quad E\,I\,\frac{\partial^2 y}{\partial x^2} = 0$

$$-\,E\,I\,\frac{\partial^3 y}{\partial x^3} = -\,P(t) = -\,P\,e^{i\omega t}$$

$$x = L \qquad y = 0$$

$$\frac{\partial y}{\partial x} = 0 \qquad \ldots . \quad (3.5)$$

(3.3) <u>Non-dimensionalisation of the equation</u>

$$\frac{\partial^4 y}{\partial x^4} + \frac{m_o}{EI}\frac{\partial^2 y}{\partial t^2} + \frac{k}{EI}\,y = 0$$

Expressing Equn. (3.5) in a non-dimensional form

$$\frac{\partial^4 A_y}{\partial z^4} + m_o\,\frac{\partial^2 A_y}{\partial t'^2} + r_o\,A_y = 0 \quad \ldots \quad (3.8)$$

where, $A_y = \dfrac{EI\,y}{P\,T^3}$

m_o = mass ratio

$$= \frac{m}{EI}\,\frac{T^4}{T_o^{\,2}}$$

r_o = stiffness ratio

$$= \frac{k\,T^4}{EI}$$

$$Z = \frac{x}{T}$$

$$T = \sqrt[5]{\frac{E\,I}{n_h}} = \text{relative stiffiness factor.}$$

$$t' = \frac{t}{T_o}$$

T_o= Time factor

The boundary and initial conditions can be written as

<u>CASE I</u>:

$$Z = 0 \qquad \frac{\partial^2 A_y}{\partial Z^2} = 0$$

$$\frac{\partial^3 A_y}{\partial Z^3} = - e^{i\omega' t'}$$

$$Z = Z_{max} \qquad \frac{\partial^2 A_y}{\partial Z^2} = 0 \qquad \ldots\ldots (3.9)$$

$$\frac{\partial^3 A_y}{\partial Z^3} = 0$$

<u>CASE 2</u>

$$Z = 0 \qquad \frac{\partial^2 A_y}{\partial Z^2} = 0$$

$$\frac{\partial^3 A_y}{\partial Z^3} = - e^{i\omega' t'} \qquad \ldots\ldots (3.10)$$

$$Z = Z_{max} \qquad A_y = 0$$

$$\frac{\partial A_y}{\partial Z} = 0$$

(3.4) <u>Solution</u>:

Solving Equation (3.5) by variable seperable method,

Let $A_y = \phi(Z)\, e^{i\ \ t'}$ . . . (3.11)

Substituting Equation (3.11) in Equation (3.5) and cancelling $e^{i\omega' t'}$ throughout,

$$\frac{d^4\phi}{dZ^4} - m_o \omega'^2 \phi(Z) + r_o\ \phi(Z) = 0$$

$$\frac{d^4\phi}{dZ^4} + (r_o - m_o \omega'^2)\ \phi(Z) = 0$$

$$\phi(Z) = A\ \text{Sin}\,\beta Z\ \text{Sinh}\,\beta Z + B\ \text{Sin}\,\beta Z\ \text{Cosh}\,\beta Z + C\ \text{Cos}\,\beta Z\ \text{Sinh}\,\beta Z + D\ \text{Cosh}\,\beta Z\ \text{Cos}\,\beta Z \quad \ldots\ldots (3.12)$$

$$A_y = (A\ \text{Sin}\,\beta Z\ \text{Sinh}\,\beta Z + B\ \text{Sin}\,\beta Z\ \text{Cosh}\,\beta Z + C\ \text{Cos}\,\beta Z\ \text{Sinh}\,\beta Z + D\ \text{Cos}\,\beta Z\ \text{Cosh}\,\beta Z)\, e^{i\omega' t'} \quad \ldots (3.13)$$

where $\beta = \sqrt[4]{\dfrac{r_o - m_o \omega'^2}{4}}$

Using the boundary conditions the following symultaneous equations are arrived at.

<u>CASE I</u>: $A = 0$ . . (3.13)

$2B - 2C = -1/\beta^3$ . . . . (3.15)

$B\ \text{Cos}\ f_1\ \text{Sinh}\ f_1 - C\ \text{Sin}\ f_1\ \text{Cosh}\ f_1 - D\ \text{Sin}\ f_1\ \text{Sinh}\ f_1 = 0$ . . . (3.16)

$B(\text{Cos}\ f_1\ \text{Cosh}\ f_1 - \text{Sin}\ f_1\ \text{Sinh}\ f_1)$
$+C(-\text{Cos}\ f_1\ \text{Cosh}\ f_1 - \text{Sin}\ f_1\ \text{Sinh}\ f_1)$
$+D(-\text{Cos}\ f_1\ \text{Sinh}\ f_1 - \text{Sin}\ f_1\ \text{Cosh}\ f_1) = 0$ . . (3.17)

CASE 2:

$$A = 0 \qquad \ldots\ldots (3.18)$$

$$2B - 2C = -1/\beta^3 \qquad \ldots\ldots (3.19)$$

$$B(\sin f_1 \cosh f_1 + C \cos f_1 \sinh f_1$$
$$+ D \cos f_1 \cosh f_1 = 0 \quad \ldots \quad (3.20)$$

$$B(\cos f_1 \cosh f_1 + \sin f_1 \sinh f_1)$$
$$+ C(-\sin f_1 \sin f_1 + \cos f_1 \cosh f_1)$$
$$+ D(-\sin f_1 \cosh f_1 + \cos f_1 \sinh f_1) = 0$$
$$\ldots (3.21)$$

where $f_1 = \beta Z_{max}$

The equations are solved by the matrix inversion method and the values of the constants are arrived at. The deflection, bending moment shear force, and soil pressure are calculated for various values of m_o, r_o, Z_{max} and ω and the results are given in Figs. (3.1) to (3.4).

3.5 DISCUSSION OF RESULTS AND CONCLUSION.

The variation of deflection, bending moment, shear force and soil pressure along the length are seen clearly from Fig. (3.1) to (3.4) for cases (1) and (2). When ω and m_o and r_o are varied the above quantities vary by only a small extent since the value of β which is equal to $\sqrt[4]{r_o - \omega^2 m_o}$, is not

much affected by the variations in ω and m_o and r_o as is implied in the pseudo-static approach, the behaviour of the pile in the dynamic case is very similar to its behaviour under static conditions. Large values of m_o and ω_o would be causing considerable difference in the magnitude of the pile characteristics. Alternately an equivalent coefficient of subgrade reaction r_{eq} can be defined as

$$r_{eq} = r_o - m_o \omega^2 \quad . . . . \quad (3.22)$$

Eqn. (3.22) implies that the dynamic behaviour of the pile with r_o, m_o and ω is equivalent to the static behaviour of the pile with r_{eq}. Hence the dynamic behaviour of a laterally loaded pile can be predicted by an equivalent statically loaded pile.

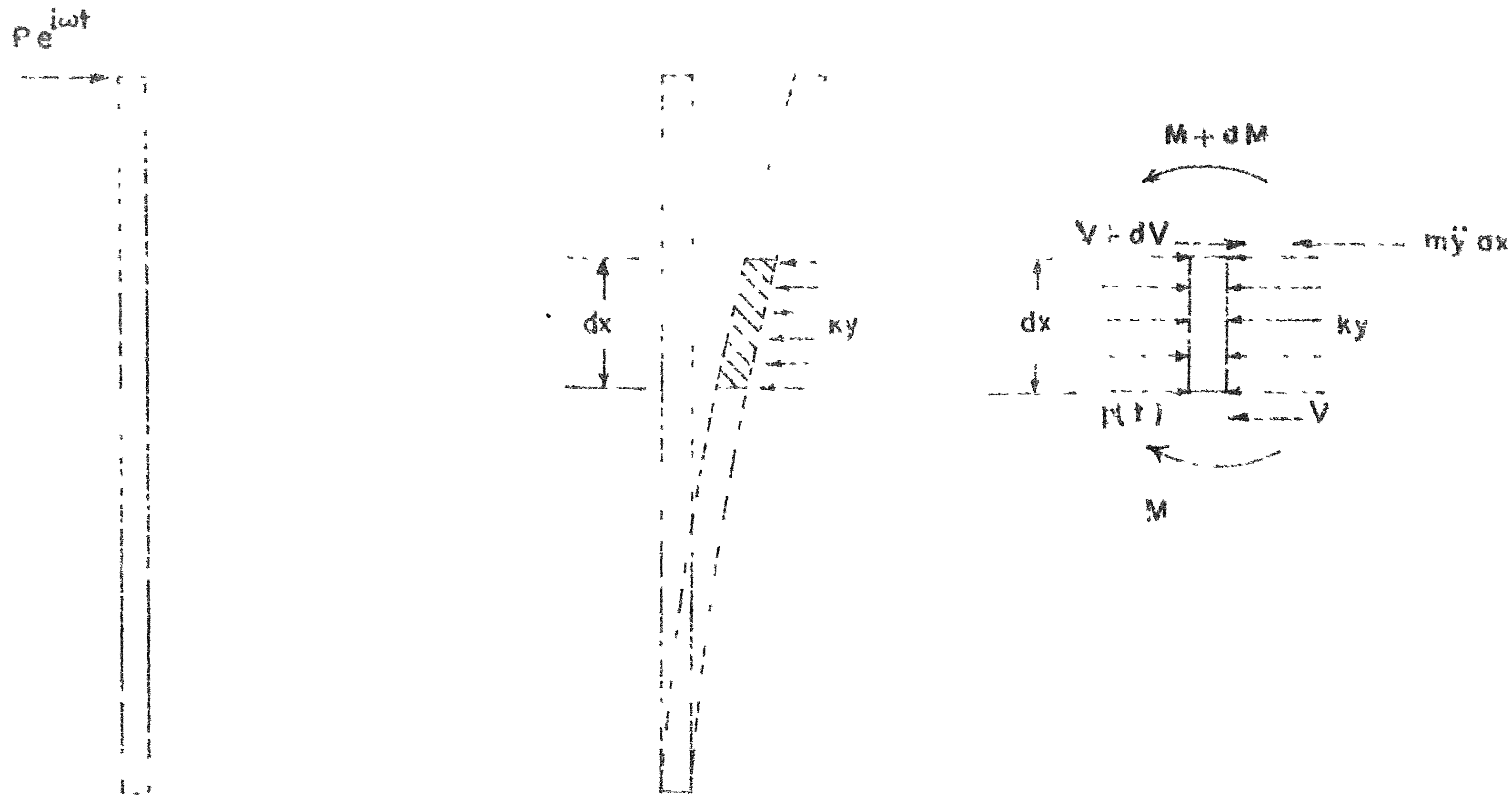
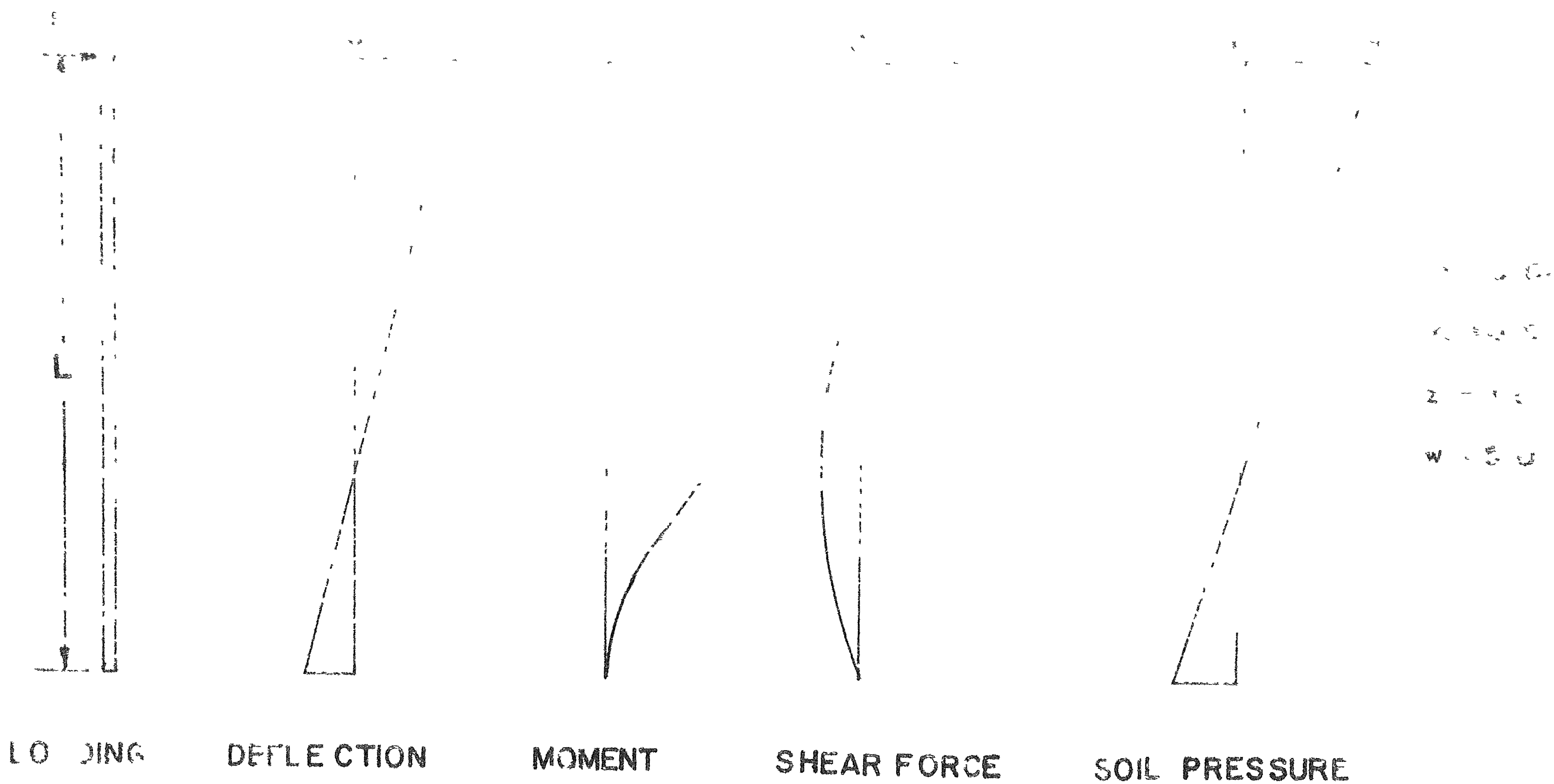
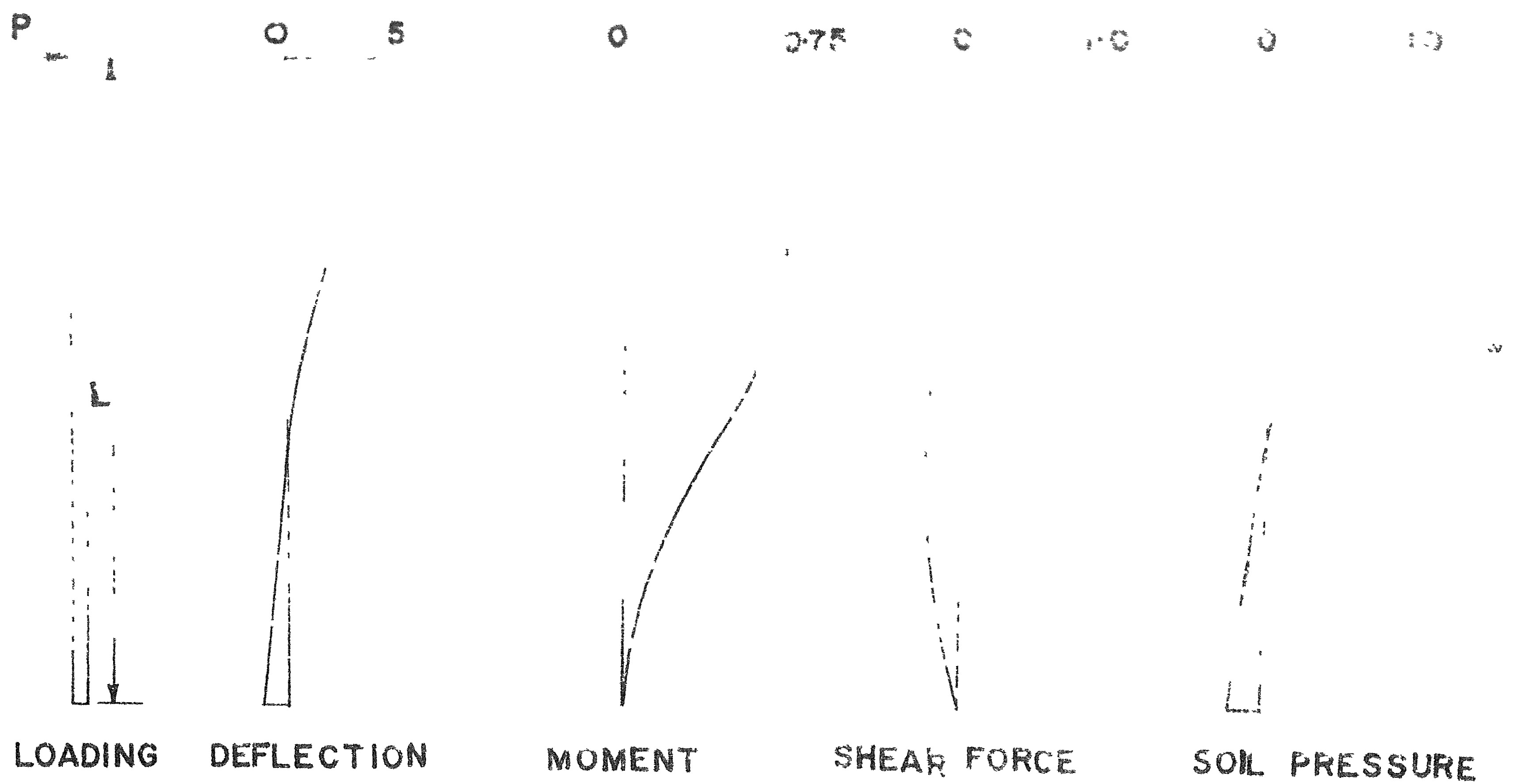


FIG (3·0) DIAGRAMATIC SKETCH OF THE PILE



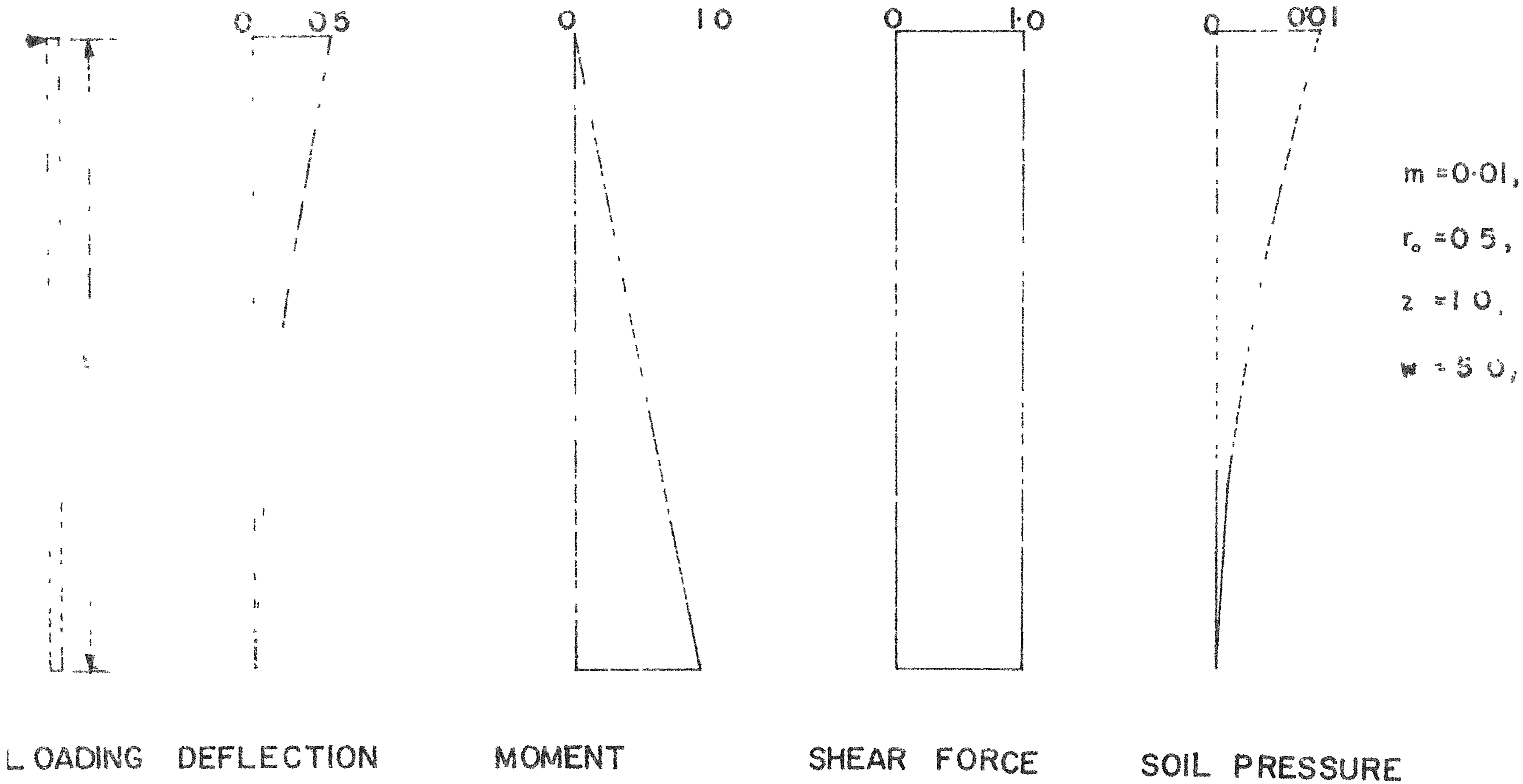
CASE I SHORT PILE

FIG (3·1) VARIATION OF DEFLECTION, MOMENT, SHEAR FORCE & SOIL PRESSURE WITH LENGTH



CASE I LONG PILE

FIG (3·2) VARIATION OF DEFLECTION, MOMENT, SHEAR FORCE & SOIL PRESSURE WITH LENGTH



CASE 2 SHORT PILE

FIG (3·3) VARIATION OF DEFLECTION MOMENT, SHEAR FORCE & SOIL PRESSURE WITH LENGT

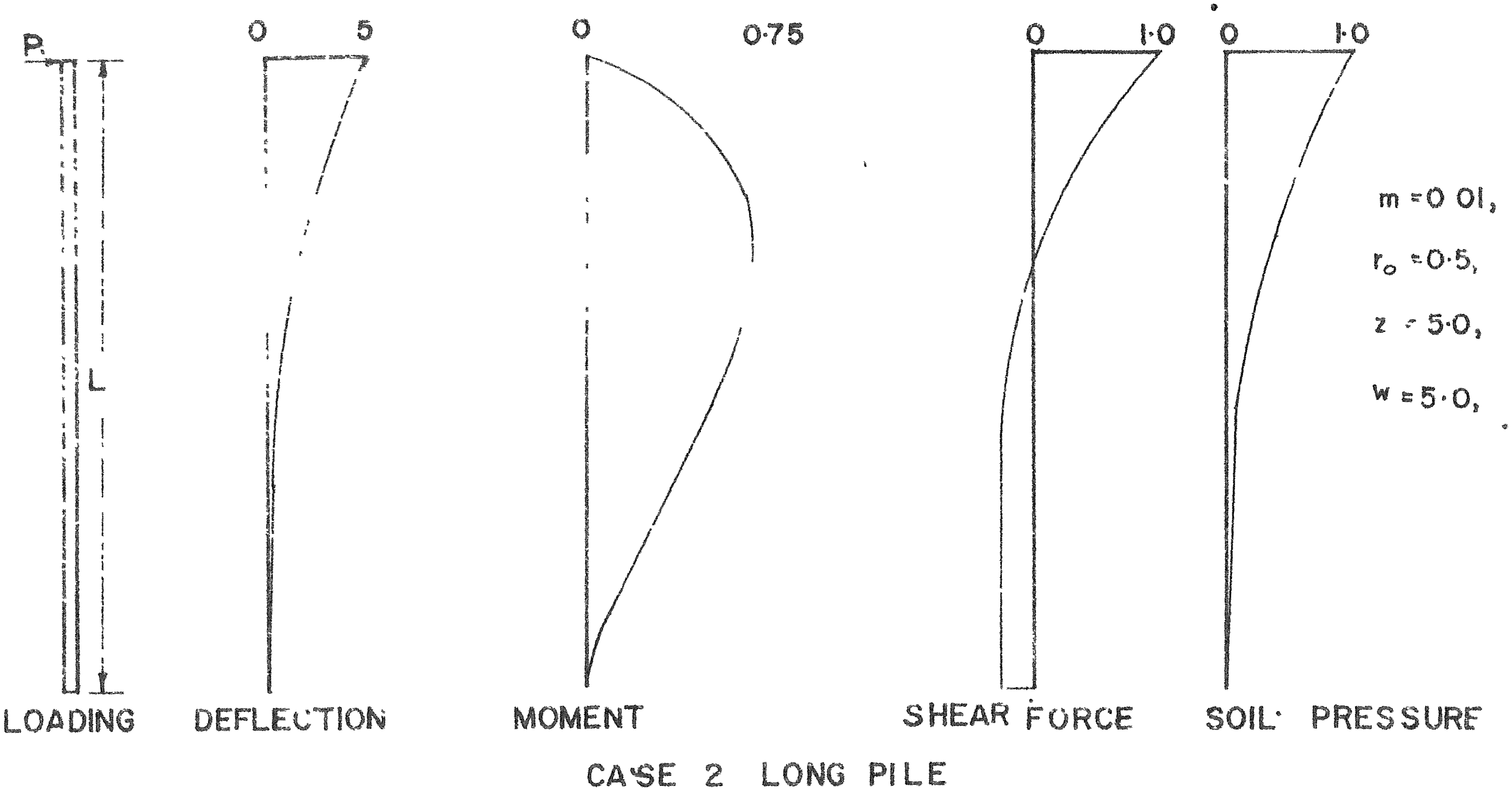


Fig (3·4) VARIATION OF DEFLECTION, MOMENT, SHEAR FORCE & SOIL PRESSURE WITH LENGTH

CHAPTER IV

LATERALLY LOADED PILE IN ELASTO-PLASTIC SOIL

4.1 INTRODUCTION

Very often in analytical solutions the soil is assumed to behave elastically under lateral loading. But in most cases it is more likely that atleast the soil in the top few feet will be in the plastic range due to the large deflections at the top of the pile. An analysis is presented for the case where the soil behaves as an elasto-plastic material.

The stress-strain behaviour of a soil is generally non-linear and is commonly described by a hyperbolic (curve (a) Fig. 4.1) type of stress-strain response (Kondan, 1963). The actual relationship unfortunately can not be included in rigorously solving for the response of a pile in such a soil. It is felt reasonable to idealise the behaviour by the curve (b) in Fig. (4.1). The idealised curve consists of a linear part upto a stress level q, wherein the ratio of stress to strain or to the displacement is a constant leading to Winkler type of foundation. The soil can not take up stress higher than q and will plastically flow. The total behaviour can be described as elasto-plastic. Mathematically curve (b) of Fig. (4.1) can be expressed as

$$\left.\begin{aligned} \sigma &= K'\epsilon \quad \text{or} \\ &= ky \end{aligned}\right\} \quad \text{if} \quad y \leq q/k$$

$$\text{and} \quad = q \qquad\qquad \text{if} \quad y > q/k$$

where K' is the modulus of deformation, k is the coefficient of subgrade reaction, y is displacement, ϵ is strain, σ - stress and q - the yield stress or ultimate strength of the soil.

4.2 FORMULATION OF THE PROBLEM:

Considering the pile as a beam on elastic foundation as explained before, the basic differential equation governing the deflection can be written as

$$EI \frac{d^4y}{dx^4} = -p \qquad \ldots\ldots \quad (4.1)$$

Here the soil pressure p is that at the limiting stage for the top portion and is denoted as 'q' and that for the bottom portion, it is ky where k is assumed to be constant throughout the length of the pile

$$EI \frac{d^4y_1}{dx^4} = -q \quad \text{in } 0 \leq x \leq x_1 \quad \ldots \quad (4.2)$$

$$EI \frac{d^4y_2}{dx^4} = -ky \quad \text{in } x_1 \leq x \leq L \quad \ldots \quad (4.3)$$

where x_1 is the point at which the soil behaviour

changes from plastic to elastic, y1 and y2 are the deflections in the top and bottom portions respectively.

Now equation (4.2) and (4.3) are to be solved to get the solutions with the help of the boundary conditions. The boundary conditions considered here are:

CASE 1 TOP FREE AND BOTTOM HINGED.

$$x=0 \quad \text{Bending Moment} = 0 \text{ i.e. } EI\frac{d^2y1}{dx^2} = 0$$

$$\text{Shear Force} = -P \text{ i.e. } -EI\frac{d^3y1}{dx^3} = -P$$

$$x=L \quad \text{Bending Moment} = 0 \text{ i.e. } EI\frac{d^2y2}{dx^2} = 0 \tag{4.4}$$

$$\text{Shear Force} = 0 \text{ i.e. } EI\frac{d^3y2}{dx^3} = 0$$

For continuity of deflection, slope, bending moment (B.M.) and shear force (S.F.) of the pile at $x = x_1$, four more conditions are arrived at.

CASE 2 : TOP FREE AND BOTTOM FIXED.

$$x=0 \quad B.M. = 0 \text{ i.e. } EI\frac{d^2y1}{dx^2} = 0$$

$$S.F. = 0 \text{ i.e. } -EI\frac{d^3y1}{dx^3} = P$$

$$x = L \quad \text{Deflection}=0 \text{ i.e. } y2 = 0 \tag{4.5}$$

$$\text{Slope} = 0 \text{ i.e. } \frac{dy2}{dx} = 0$$

As in Case 1, the four additional continuity conditions are got.

4.3 NON-DIMENTIONAL ANALYSIS:

The equations can be reduced to a non-dimensional form by the principle of dimensional analysis between the different variables of the laterally loaded pile. The variables involved in this problem are x, L, k, EI, P, y, q and T. So the non-dimensional solution can be expressed as

$$\frac{y}{P} = f\ (x,\ L,\ k,\ T,\ EI)$$

There are six variables and length and force are the only two dimensions. By using the Buckingham π theorem the non-dimensional terms are arrived at as $\frac{yEI}{PT^3}$, $\frac{x}{T}$, $\frac{L}{T}$, $\frac{kT^4}{EI}$ and $\frac{qT}{P_t}$ are the non-dimensional terms. The first four non-dimensional terms were given by Reese and Matlock. They denoted the first three terms as A_y, Z and Z_{max} respectively.

Using these non-dimensional terms, the equations (4.2) and (4.3) can be non-dimensionalised as follows:

$$\frac{d^4y1}{dx^4} + \frac{q}{EI} = 0 \qquad \text{in } 0 \leq x \leq x_1$$

$$\frac{PT^3}{EI}\ \frac{1}{T^4}\ \frac{d^4A_{y1}}{dz^4} + \frac{q}{EI} = 0 \text{ since } A_y = \frac{y}{P}\ \frac{EI}{T^3}$$

Denoting $\frac{qT}{P_t}$ by 'm',

$$\frac{d^4A_{y1}}{dz^4} + m = 0 \quad \text{in } 0 \leq z \leq \alpha Z_{max} \quad \ldots \quad (4.6)$$

where α is the ratio of Z_1 to Z_{max}, Z being the depth at which the soil behaviour changes from plastic to elastic.

$$\frac{d^4y2}{dx^4} + \frac{ky2}{EI} = 0 \quad \text{in } x_1 \leq x \leq L$$

$$\frac{PT^3}{EI} \quad \frac{d^4A_{y2}}{dZ^4} \quad \frac{1}{T^4} + \frac{k}{EI} \quad \frac{PT^3}{EI} \quad A_{y2} = 0$$

$$\frac{d^4A_{y2}}{dZ^4} + \frac{k\ T^4}{EI} \quad A_{y2} = 0$$

Denoting $\frac{k\ T^4}{EI}$ 'r_o',

$$\frac{d^4A_{y2}}{dZ^4} + r_o \quad A_{y2} = 0 \quad \text{in } \alpha Z_{max} \leq Z \leq Z_{max} \quad \ldots (4.7)$$

Nondimentionalising the boundary conditions,

CASE 1

$$x=0 \quad \frac{EId^2y1}{dx^2} = 0 \quad \text{i.e. } Z=0 \cdot \frac{d^2A_{y1}}{dZ^2} = 0$$

$$-EI \frac{d^3y1}{dx^3} = -P \text{ i.e. } -EI \quad \frac{P_tT^3}{EI} \quad \frac{1}{T^3} \quad \frac{d^3A_{y1}}{dZ^3} = -P$$

$$\text{i.e.} \quad \frac{d^3A_{y1}}{dZ^3} = \frac{P}{P_t} = R$$

where P_t is the ultimate load in the case of the hinged bottom piles and R is the load ratio. . . . (4.4.1)

$x=L \quad \frac{d^2y2}{dx^2} = 0 \quad$ i.e. $\quad Z = Z_{max} \quad \frac{d^2A_{y2}}{dZ^2} = 0$,

$\frac{d^3y2}{dx3} = 0 \quad$ i.e. $\quad \frac{d^3A_{y2}}{dZ^3} = 0$

CASE 2

$$Z=0 \quad \frac{d^2A_{y1}}{dz^2} = 0$$

$$\frac{d^3A_{y1}}{dZ^3} = R \qquad \ldots\ldots\ldots\ldots (4.5.1)$$

$$Z=Z_{max} \quad A_{y2} = 0$$

$$\frac{dA_{y2}}{dZ} = 0$$

The Equations (4.6) and (4.7) are to be solved by using the boundary conditions (4.4.1) and (4.5.1)

4.4 SOLUTION·

(1) Plastic Region:

$$\frac{d^4A_{y1}}{dZ^4} = -m \qquad \ldots\ldots (4.8.4)$$

$$\frac{d^3A_{y1}}{dZ^3} = -mZ + C_1 \qquad \ldots\ldots (4.8.3)$$

$$\frac{d^2A_{y1}}{dZ^2} = -\frac{mZ^2}{2} + C_1 Z + C_2 \qquad \ldots (4.8.2)$$

$$\frac{dA_{y1}}{dZ} = -\frac{m Z^3}{6} + C_1 \frac{Z^2}{2} + C_2 Z + C_3 \quad . (4.8.1)$$

$$A_{y1} = -m\frac{Z^4}{24} + C_1\frac{Z^3}{6} + C_2\frac{Z^2}{2} + C_3 Z + C_4 \quad \ldots (4.8)$$

(2) Elastic Region:

$$\frac{d^4A_{y2}}{dZ^4} + r_o A_{y2} = 0$$

$$A_{y2} = A \operatorname{Sin} f_1 \operatorname{Sinh} f_1 + B \operatorname{Sin} f_1 \operatorname{Cosh} f_1 + C \operatorname{Cos} f_1 \operatorname{Sinh} f_1 + D \operatorname{Cos} f_1 \operatorname{Cosh} f_1 \quad \ldots (4.9)$$

where $f_1 = \beta Z$ and $\beta = \sqrt[4]{r_o/4}$

$$\frac{dA_{y2}}{dZ} = \beta\left[A(\operatorname{Cos} f_1 \operatorname{Sinh} f_1 + \operatorname{Sin} f_1 \operatorname{Cosh} f_1) + B(\operatorname{Cos} f_1 \operatorname{Cosh} f_1 + \operatorname{Sin} f_1 \operatorname{Sinh} f_1) + C(-\operatorname{Sin} f_1 \operatorname{Sinh} f_1 + \operatorname{Cos} f_1 \operatorname{Cosh} f_1) + D(-\operatorname{Sin} f_1 \operatorname{Cosh} f_1 + \operatorname{Cos} f_1 \operatorname{Sinh} f_1)\right] \quad \ldots (.4.9.1)$$

$$\frac{d^2A_{y2}}{dZ^2} = \beta^2\left[A(2 \operatorname{Cos} f_1 \operatorname{Cosh} f_1) + B(2 \operatorname{Cos} f_1 \operatorname{Sinh} f_1) + C(-2 \operatorname{Sin} f_1 \operatorname{Cosh} f_1) + D(-2 \operatorname{Sin} f_1 \operatorname{Sinh} f_1)\right] \quad \ldots (4.9.2)$$

$$\frac{d^3A_{y2}}{dZ^3} = \beta^3\left[A(2 \operatorname{Cos} f_1 \operatorname{Sinh} f_1 - 2 \operatorname{Sin} f_1 \operatorname{Cosh} f_1) + B(2 \operatorname{Cos} f_1 \operatorname{Cosh} f_1 - 2 \operatorname{Sin} f_1 \operatorname{Sinh} f_1) + C(-2\operatorname{Cos} f_1 \operatorname{Cosh} f_1 - 2 \operatorname{Sin} f_1 \operatorname{Sinh} f_1) + D(-2\operatorname{Cos} f_1 \operatorname{Sinh} f_1 - 2 \operatorname{Sin} f_1 \operatorname{Cosh} f_1)\right] \quad \ldots (4.9.3)$$

where A, B, C, D, C_1, C_2, C_3, C_4 ~~and~~ are constants to be evaluated and the value of R i.e. P/P_t is to be given. Since eight conditions are available, eight constants can be evaluated. The value of α is dependent on the applied force at the top. However solving for α given P_t, R is difficult if not impossible. Here the value of R is arrived at by assuming the value of α and taking into account one more condition that the soil pressures at a depth $Z = \alpha Z_{max}$ in both the plastic and elastic regions are equal.

The following are the equations fullfilling all the nine conditions:

Case 1: <u>TOP FREE AND BOTTOM HINGED:</u>

$$C_1 = R \qquad \ldots\ldots\ldots \qquad (4.10.1)$$

$$C_2 = 0 \qquad \ldots\ldots\ldots \qquad (4.10.2)$$

$$A(\cos f_2 \cosh f_2) + B \cos f_2 \sinh f_2 + C(-\sin f_2 \cosh f_2) + D(-\sin f_2 \sinh f_2) = 0 \qquad \ldots\ldots \qquad (4.10.3)$$

where $f_2 = \beta Z_{max}$

$$A(\cos f_2 \sinh f_2 - \sin f_2 \cosh f_2) + B(\cos f_2 \cosh f_2 - \sin f_2 \sinh f_2) + C(-\cos f_2 \cosh f_2 - \sin f_2 \sinh f_2) + D(-\cos f_2 \sinh f_2 - \sin f_2 \cosh f_2) = 0 \qquad \ldots\ldots \qquad (4.10.4)$$

$$- m\alpha^4 \frac{Z_{max}^4}{24} + C_1 \alpha^3 \frac{Z_{max}^3}{6} + \frac{C_2}{2} \alpha^2 Z_{max}^2 + C_3 \alpha Z_{max}$$

$$+ C_4 = A \sin f_3 \sinh f_3 + B \sin f_3 \cosh f_3$$

$$+ C \cos f_3 \sinh f_3 + D \cos f_3 \cosh f_3$$

$$\ldots \quad (4.10.5)$$

where $f_3 = \alpha/\beta \; Z_{max}$

$$- m\alpha^3 Z_m^3/6 + C_1\alpha^2 Z_m^2/2 + C_2\alpha Z_m + C_3$$

$$= \beta [A(\cos f_3 \sinh f_3 + \sin f_3 \cosh f_3)$$

$$+ B(\cos f_3 \cosh f_3 + \sin f_3 \sinh f_3)$$

$$+ C(-\sin f_3 \sinh f_3 + \cos f_3 \cosh f_3)$$

$$+ D(-\sin f_3 \cosh f_3 + \cos f_3 \sinh f_3)]$$

$$\ldots \quad (4.10.6)$$

where $Z_m = Z_{max}$

$$- m\alpha^2 Z_m^2/2 + C_1\alpha Z_m + C_2$$

$$= \beta^2 [A(2 \cos f_3 \cosh f_3) + B(2 \cos f_3 \sinh f_3)$$

$$+ C(-2 \sin f_3 \cosh f_3) + D(-2 \sin f_3 \sinh f_3)]$$

$$\ldots \quad (4.10.7)$$

$$- m\alpha Z_m + C_1 = \beta^3 [A(2 \cos f_3 \sinh f_3 - 2 \sin f_3 \cosh f_3)$$

$$+ B(2 \cos f_3 \cosh f_3 - 2 \sin f_3 \sinh f_3)$$

$$+ C(-2\cos f_3 \cosh f_3 - 2 \sin f_3 \sinh f_3)$$

$$+ D(-2\cos f_3 \sinh f_3 - 2 \sin f_3 \cosh f_3)]$$

$$\ldots \quad (4.10.8)$$

$$A \;\text{Sin}\; f_3 \;\text{Sinh}\; f_3 + B \;\text{Sin}\; f_3 \;\text{Cosh}\; f_3$$
$$+ \; C \;\text{Cos}\; f_3 \;\text{Sinh}\; f_3 + D \;\text{Cos}\; f_3 \;\text{Cosh}\; f_3 = m/r_o$$

. . . (4.10.8)

Case 2 <u>TOP FREE AND BOTTOM FIXED</u>:

Equations (4.10.3) and (4.10.4) will only differ while the others remain the same.

$$A \;\text{Sin}\; f_2 \;\text{Sinh}\; f_2 + B \;\text{Sin}\; f_2 \;\text{Cosh}\; f_2$$
$$+ \; C \;\text{Cos}\; f_2 \;\text{Sinh}\; f_2 + D \;\text{Cos}\; f_2 \;\text{Cosh}\; f_2 = 0$$

. . . (4.10.3.1)

$$A(\text{Cos}\; f_2 \;\text{Sinh}\; f_2 + \text{Sin}\; f_2 \;\text{Cosh}\; f_2)$$
$$+ B(\text{Cos}\; f_2 \;\text{Cosh}\; f_2 + \text{Sin}\; f_2 \;\text{Sinh}\; f_2)$$
$$+ C(-\text{Sin}\; f_2 \text{Sinh}\; f_2 + \text{Cos}\; f_2 \;\text{Cosh}\; f_2)$$
$$+ D(-\text{Sin}\; f_2 \text{Cosh}\; f_2 + \text{Cos}\; f_2 \;\text{Sinh}\; f_2) = 0$$

. . . (4.10.4.1)

The simultaneous equations are solved by the matrix inversion method and all the values of the constants are arrived at. The values of deflection, the value of M/PT, the Shear Force factor, V/P and the soil pressure are got by substituting these constants in the corresponding equations.

The variables that are involved in these equations are m, α , r_o and Z_{max}. The solutions

are got for different values of these variables. The effect of these variables are clearly seen from the graphs drawn.

When $Z_{max} \leq 2$, the pile is called a short pile and when $Z_{max} \geqslant 5.0$, it is named as a long pile. Graphs are drawn seperately for long and short piles for cases (1) and (2). Figs. (4.2) to (4.12) represent results for case (1) and (4.13) to (4.22) represent those for case (2).

4.5 DISCUSSION OF RESULTS:

From Figs (4.2) and (4.3), it is evident that the short pile is one which behaves like a rigid pile and the long pile is one wherein the behaviour is as that of a flexible pile.

In case (1) as Z_{max} increases, the deflection for the same load ratio P/P_t decreases and this is just the reverse for the case (2) as seen in Figs. (4.4) and (4.15).

In case (1) for both short and long piles the deflection when m = 0.2 is twice its value when m = 0.5. The behaviour of the long fixed piles is similar to that of long hinged piles while different values of m do not cause significant changes in the load deflection relation of short fixed piles as seen in Figs. (4.5) and (4.16).

In case (1), the deflection is nearly halved when r_o changes from 0.2 to 0.5 as shown in Figs. (4.6) and (4.17). In the case of short piles the change in deflection is somewhat greater than that in long piles. The long piles in case (2) behave in a similar manner. 'r_o' has little effect on deflection in the case of short fixed piles.

In case (2), the deflection of short piles is many times lesser than that in case (1) as seen in Fig. (4.7). In the long piles though different boundary conditions are used at the bottom, deflections are of the same order in the two cases.

In the case of piles with hinged bottom, the deflections along the length decrease as 'Z_{max}' increases and in the case of piles with fixed bottom, the deflections along the length increase as Z_{max} increases. as seen in Figs. (4.8) and (4.18).

As [illegible] increases the deflection and bending moment increase in cases (1) and (2) as shown in Figs. (4.9), (4.19) and Figs. (4.12) and (4.22) respectively.

In cases (1) and (2), the deflection decreases along the length when 'r_o' increases, as seen in Figs. (4.10) and (4.20).

As 'm' increases the deflections increase in both cases along the depth as seen in Figs. (4.11) and (4.21).

4.6 CONCLUSION:

With the solutions developed in this chapter, the response of a laterally loaded pile in an elasto-plastic soil is presented. The effects of different parameters like the stiffness (k or r_o) and the yield stress (q or m) of the soil, the length of the pile (Z_{max}) and the depth of the plastic zone (αZ_{max}), on the load deflection relation, the deflection at the top and the moment along the length of the pile are studied. The analysis has been carried out for piles hinged or fixed at the bottom.

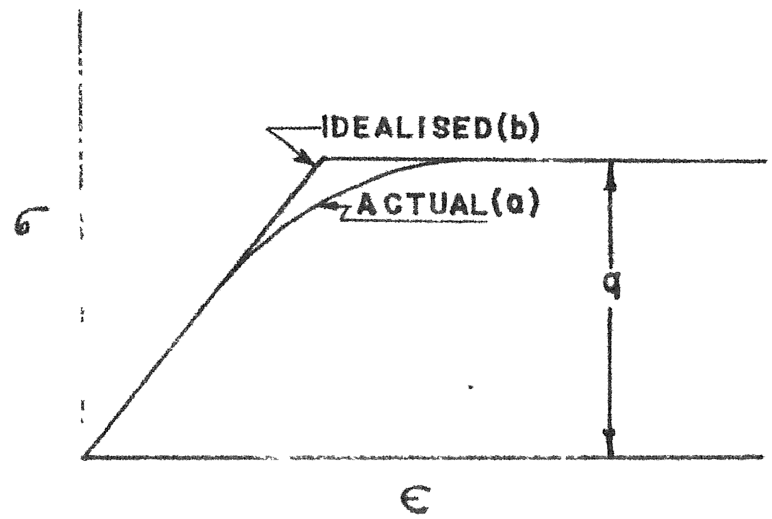
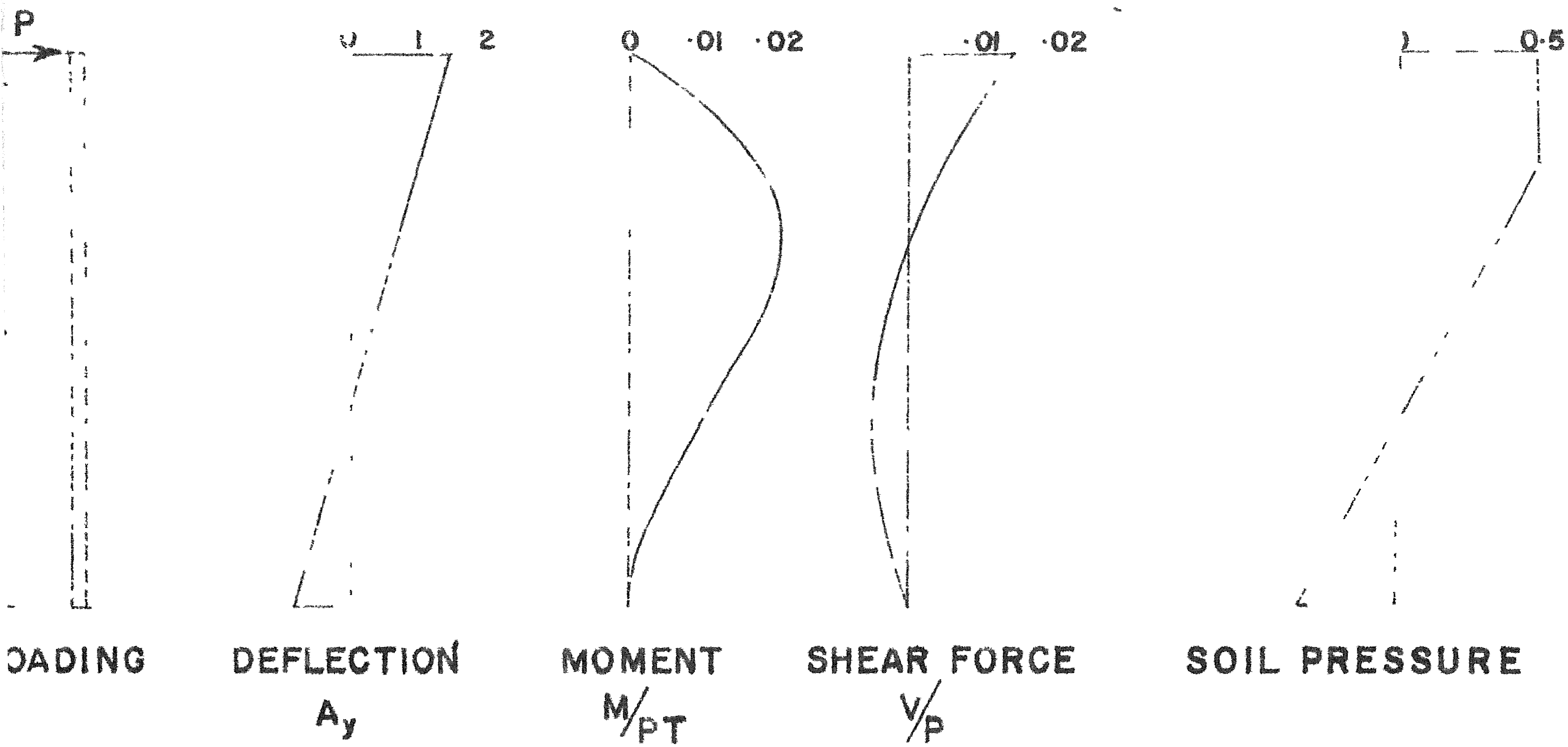
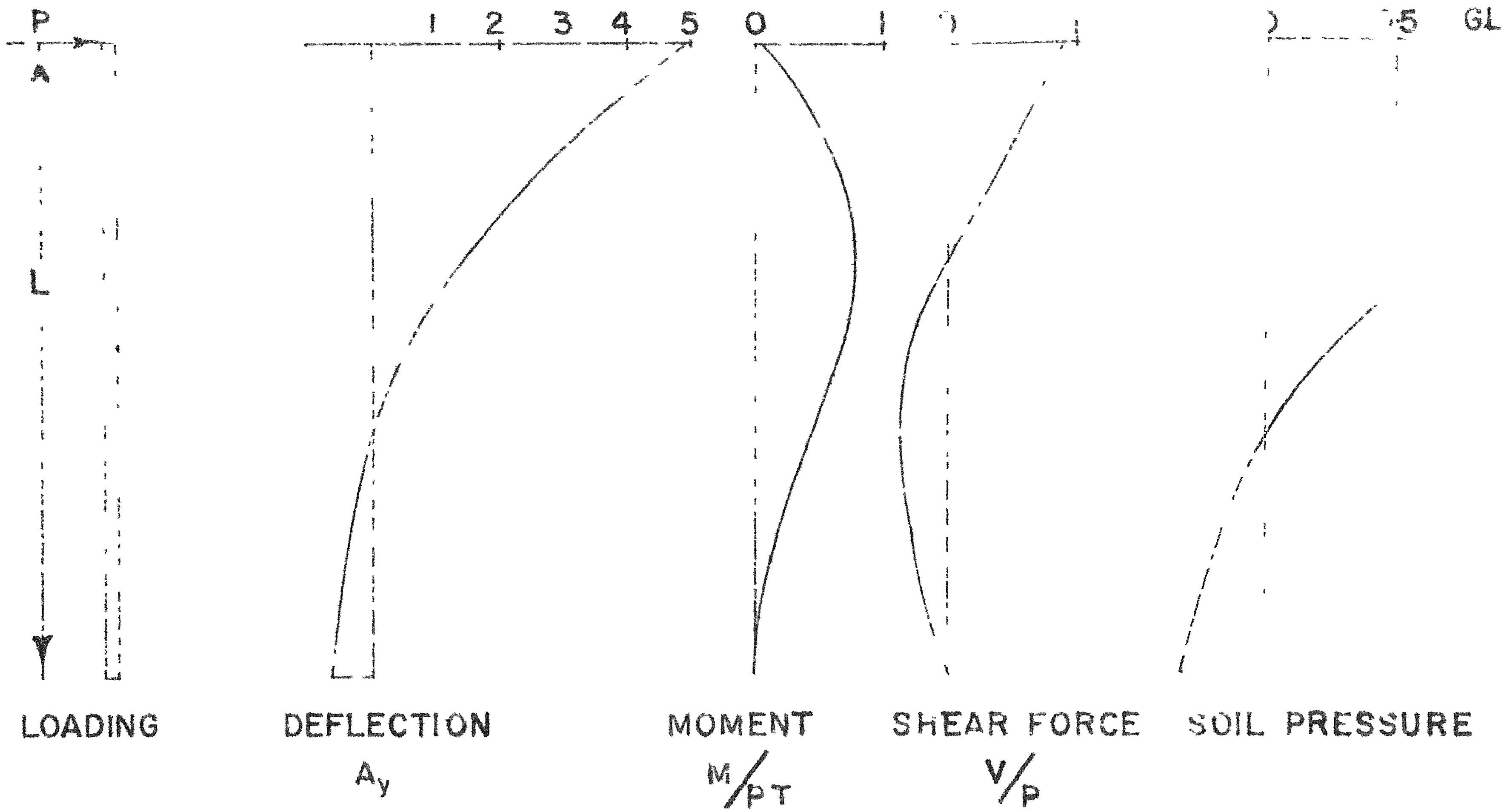


FIG (4·1) STRESS-STRAIN RESPONSE OF A SOIL



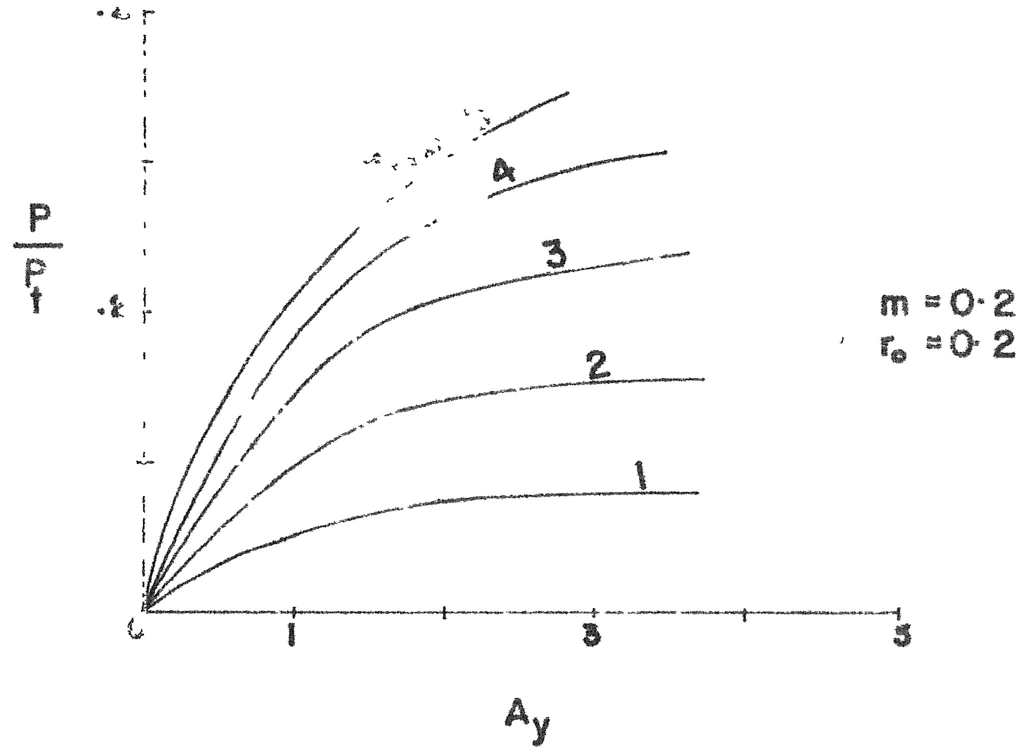
CASE : SHORT PILE

G(4·2) VARIATION OF DEFLECTION, MOMENT, SHEAR FORCE,& SOIL PRESSURE WITH LENGTH



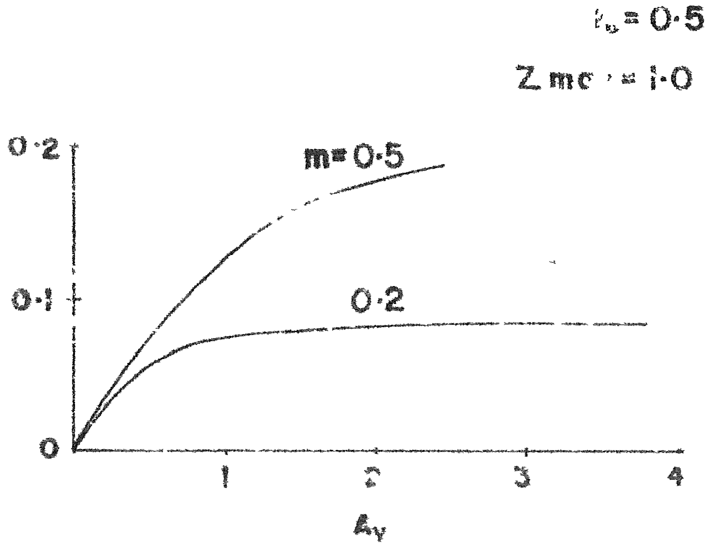
LONG PILE

FIG(4·3) VARIATION OF DEFLECTION, MOMENT, SHEAR FORCE, & SOIL PRESSURE WITH LENGTH

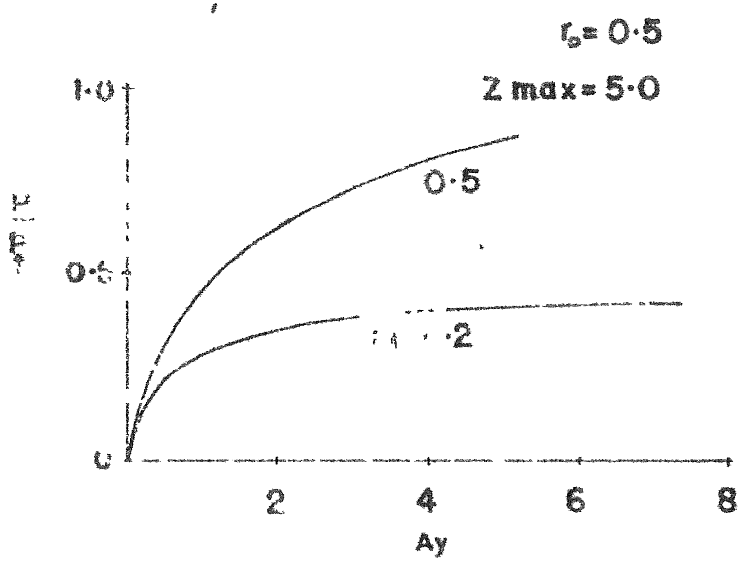


CASE I

FIG(4·4) EFFECT OF VARIATION OF Z_{max} ON LOAD DEFLECTION RELATIONSHIP



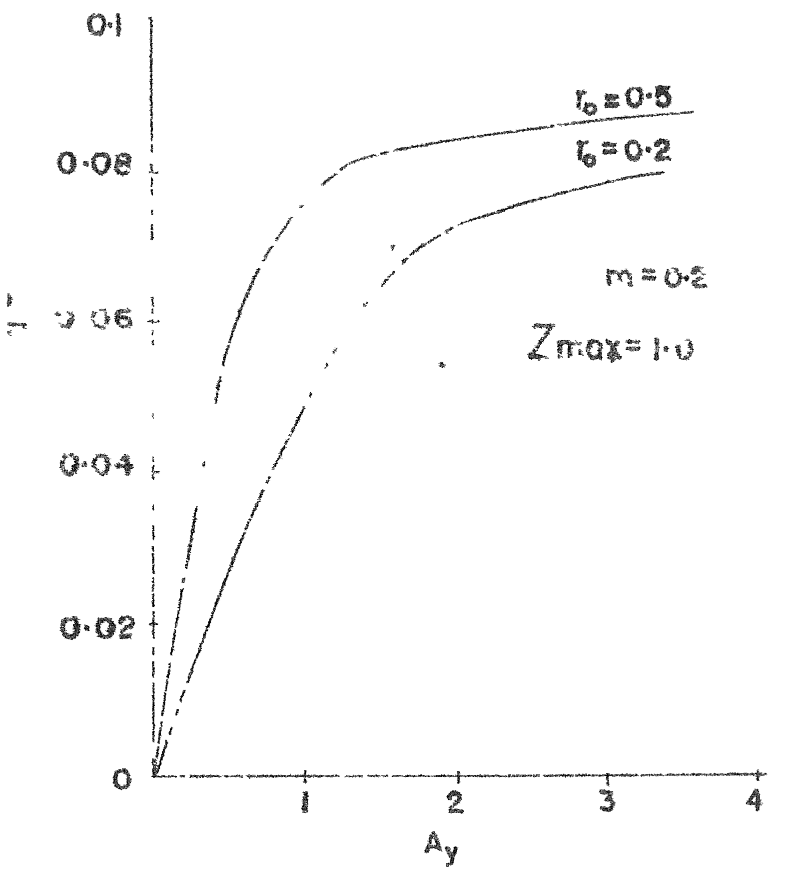
SHORT PILE



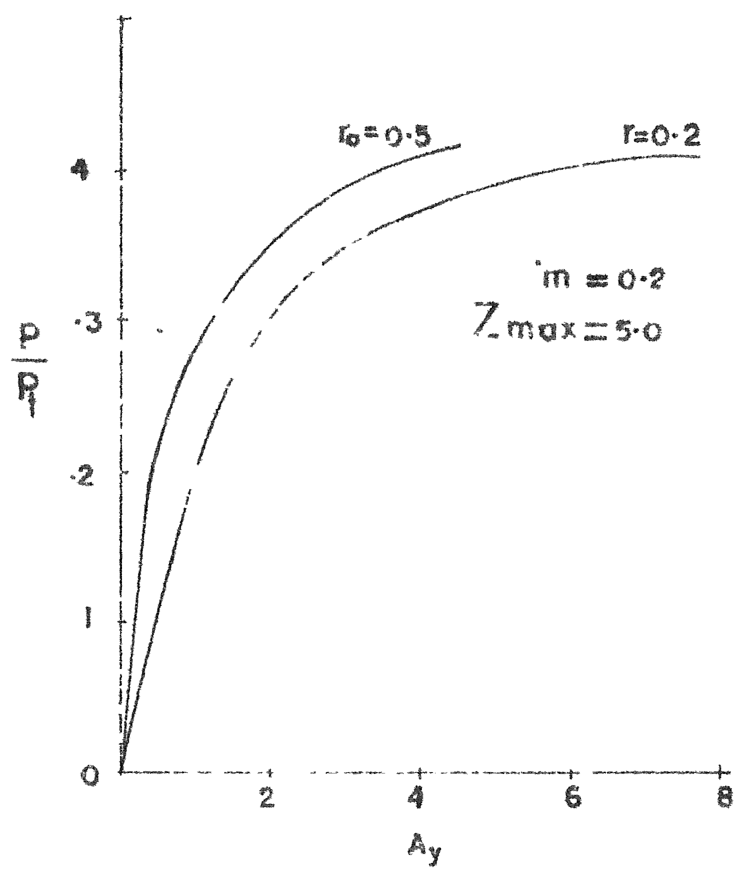
LONG PILE

CASE I

FIG(4·5) EFFECT OF m ON LOAD DEFLECTION RELATIONSHIP



SHORT PILE



LONG PILE

FIG(4·6) EFFECT OF r_o ON LOAD DEFLECTION RELATIONSHIP

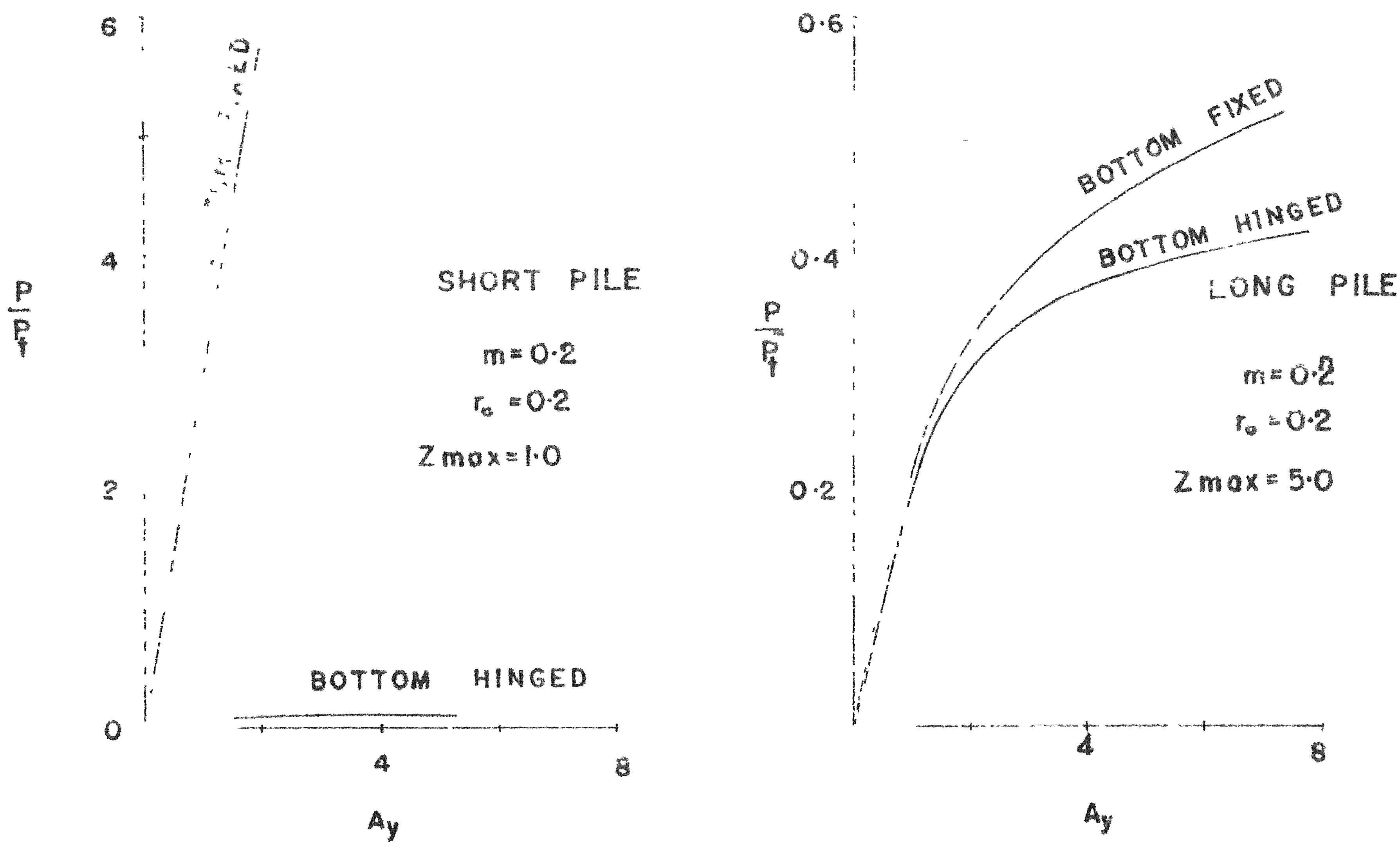
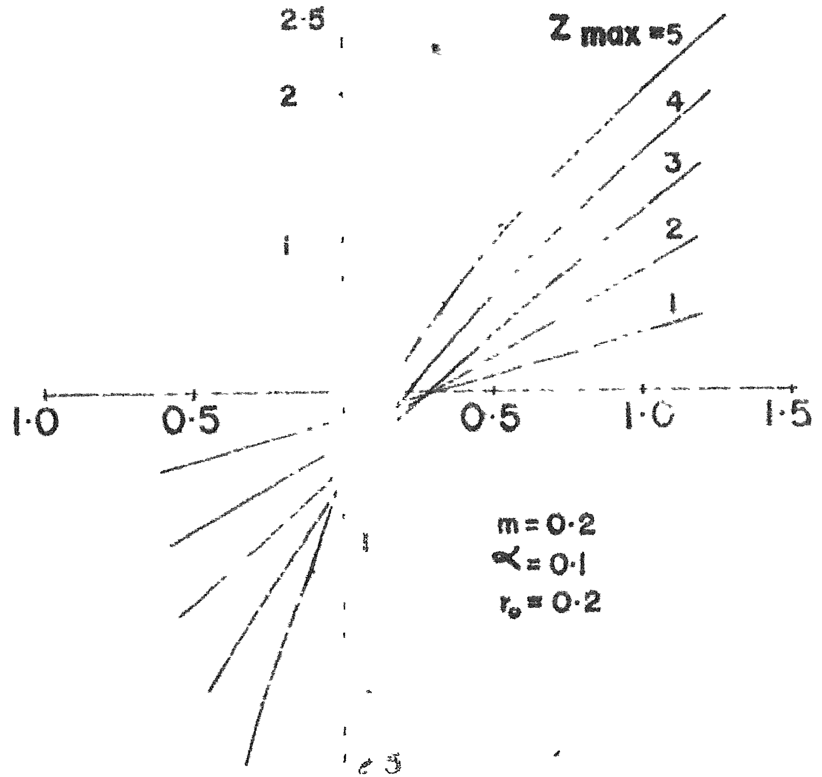


FIG (47) EFFECT OF FIXITY ON LOAD DEFLECTION RELATIONSHIP



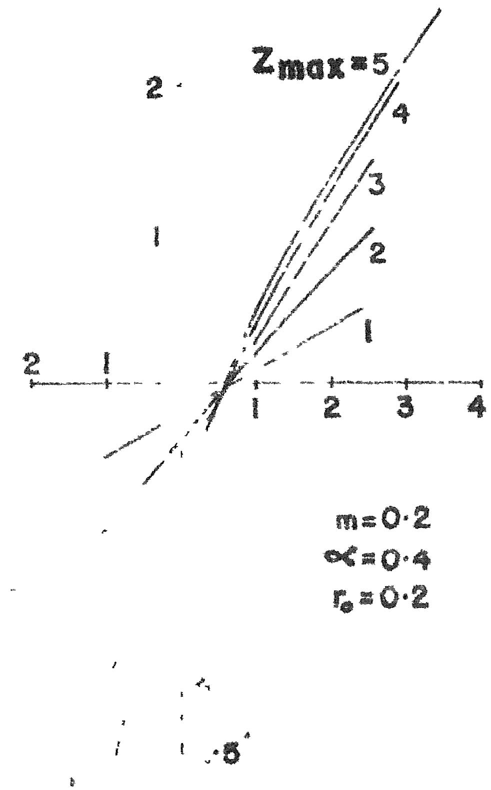
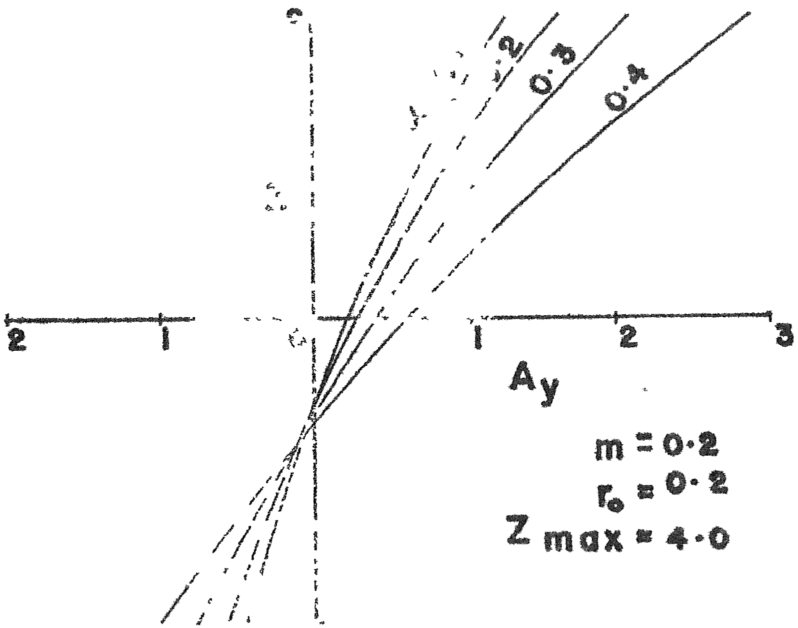
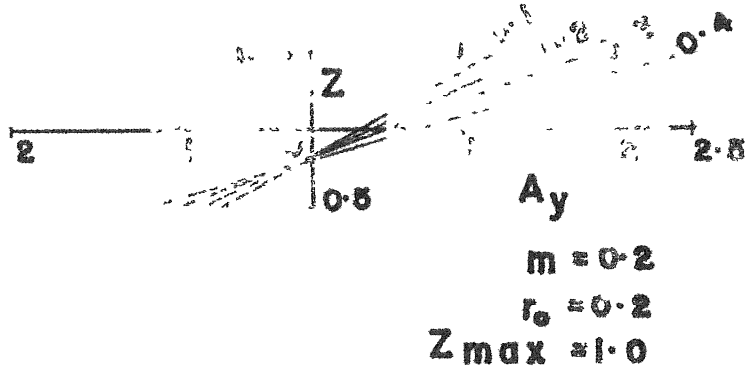


FIG (4·8) EFFECT OF Z_{max} ON DEFLECTION ALONG THE LENGTH

CASE I



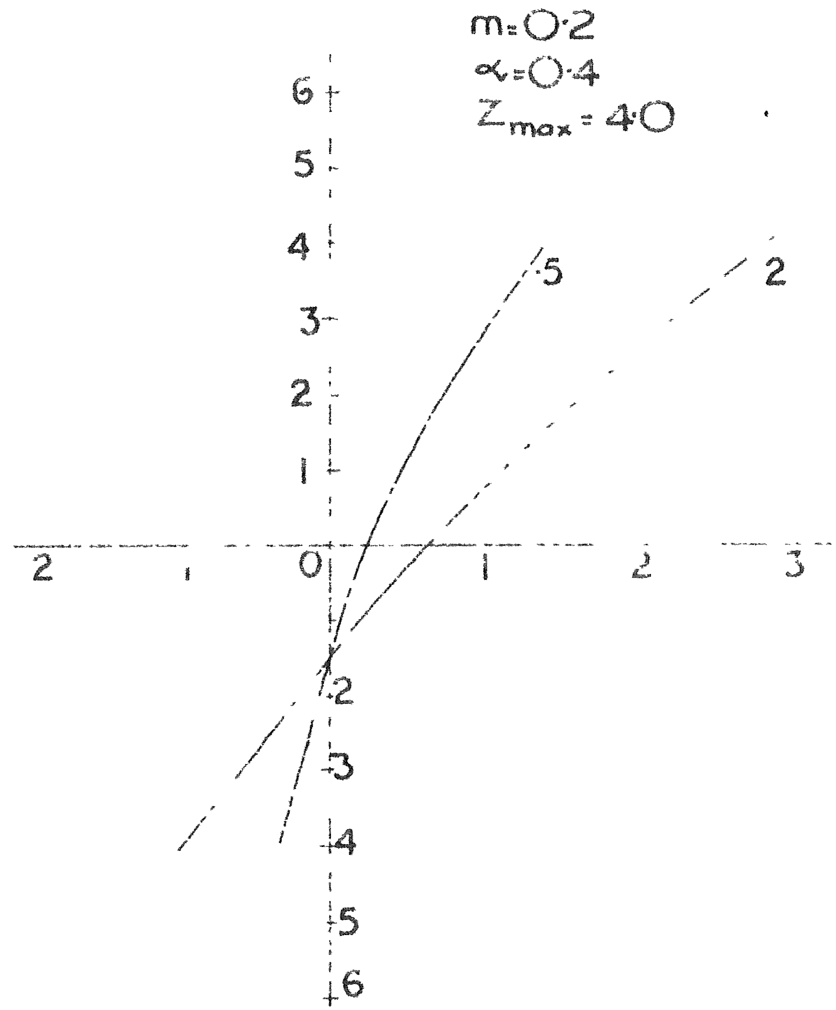
LONG PILE



SHORT PILE

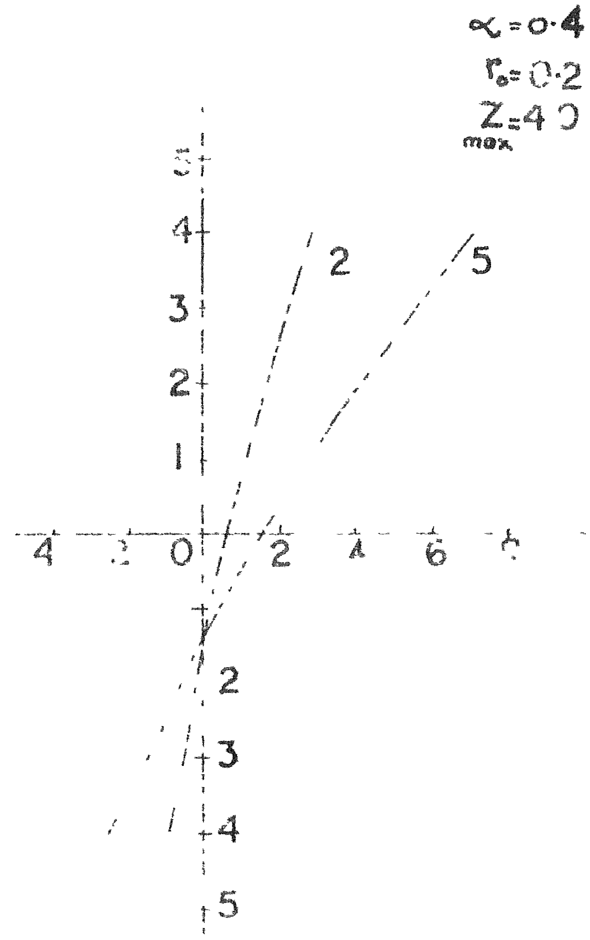
FIG(4.·) EFFECT OF $\propto$ ON DEFLECTION ALONG THE LENGTH

CASE I



LONG PILE

FIG 4.10 EFFECT OF 'r_o' ON DEFLECTION ALONG THE LENGTH

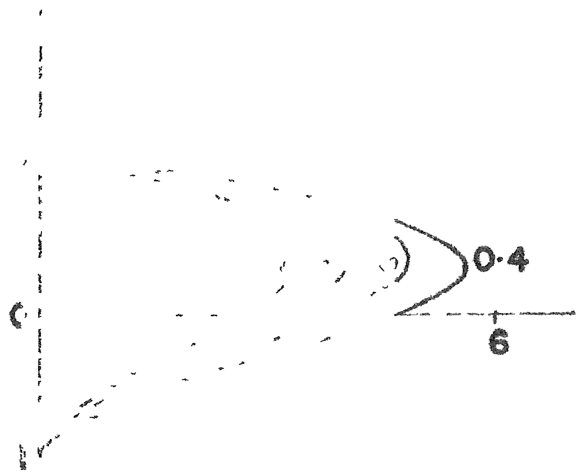


LONG PILE

FIG 4.11 EFFECT OF m ON DEFLECTION ALONG THE LENGTH

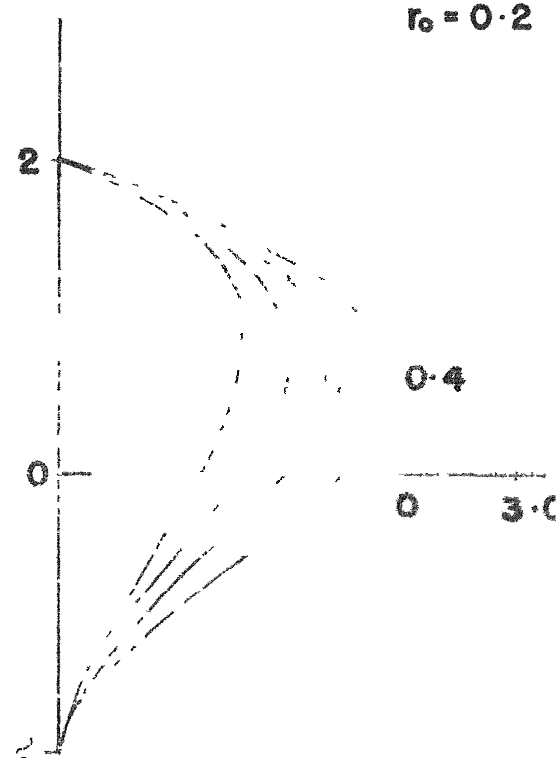
CASE I

$Z_{max} = 2.0$
$m = 0.2$
$r_o = 0.2$



SHORT PILE

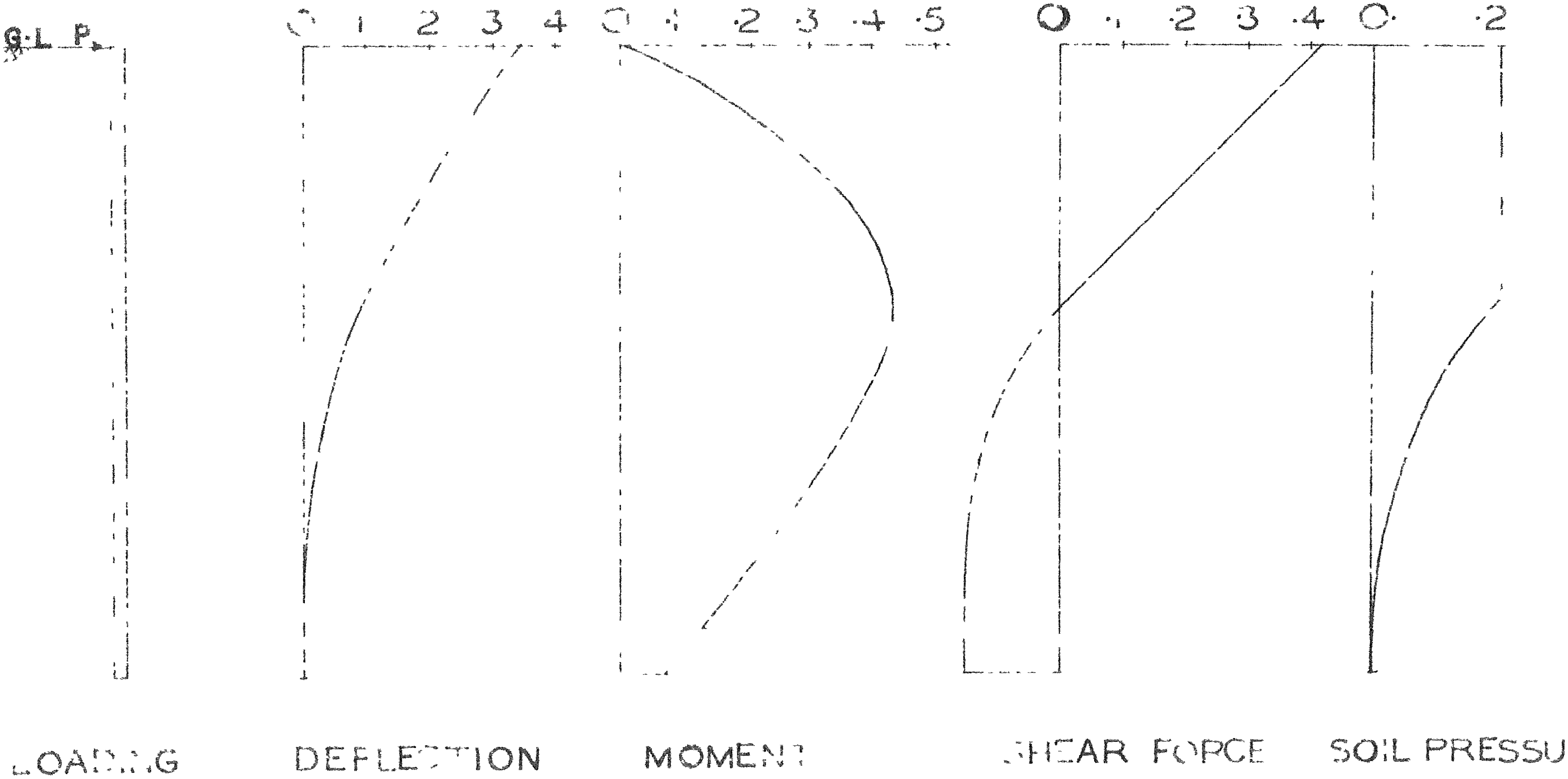
$Z_{max} = 4.0$
$m = 0.2$
$r_o = 0.2$



LONG PILE

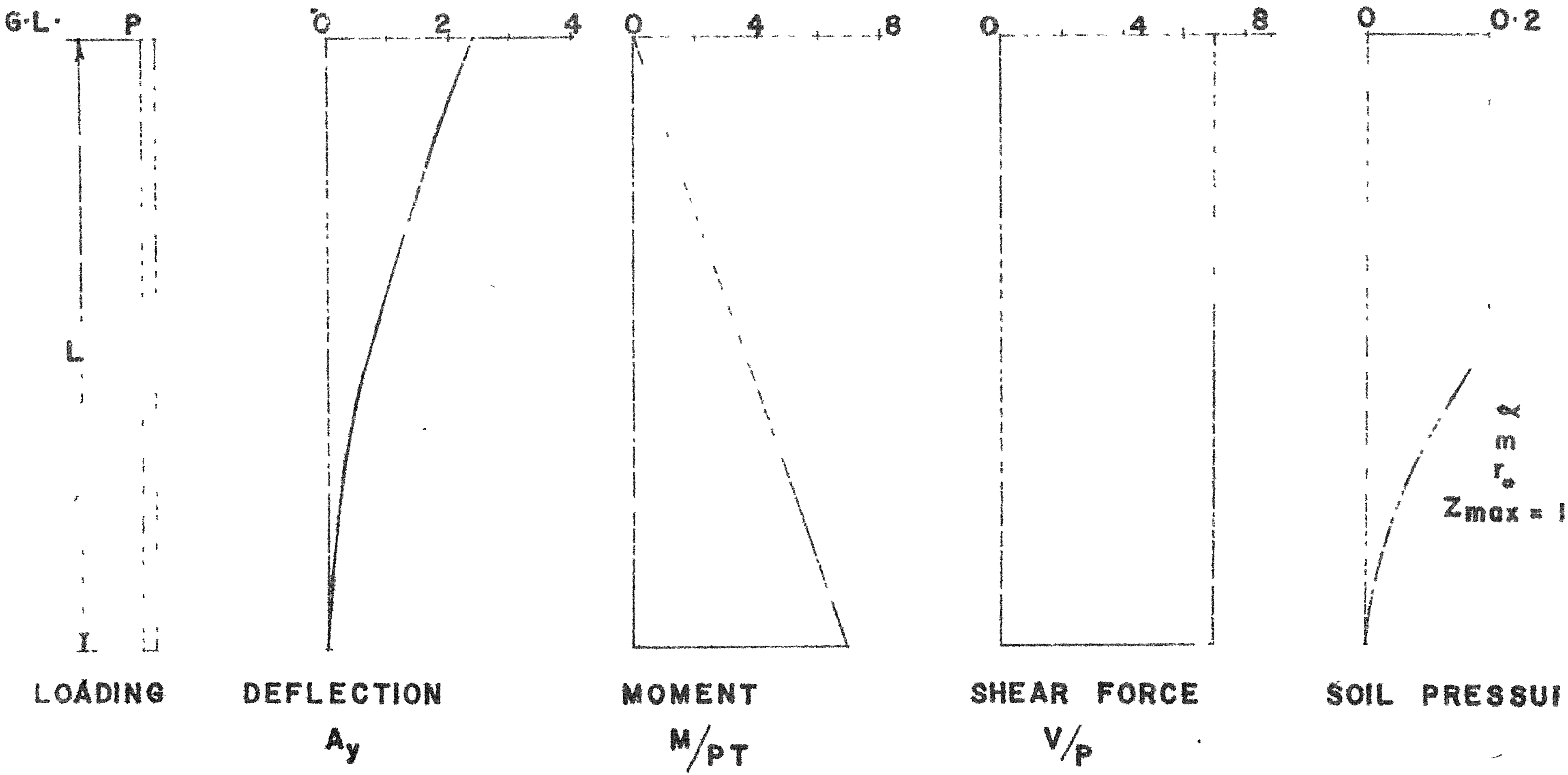
CASE I

FIG (4·12) EFFECT OF α ON BENDING MOMENT DISTRIBUTION



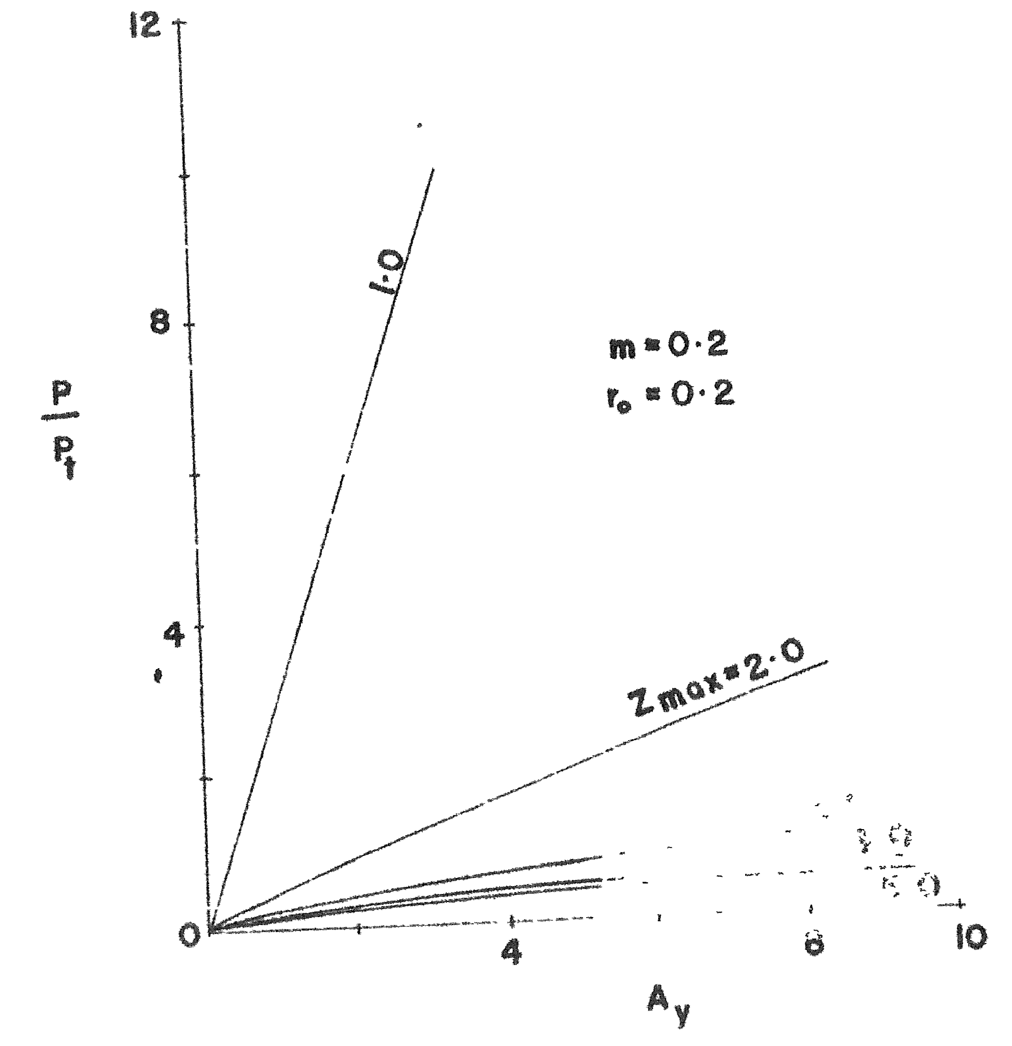
CASE 2 LONG PILE

VARIATION OF DEFLECTION, MOMENT, SHEAR FORCE & SOIL PRESSURE WITH LENGTH



CASE 2 SHORT PILE

FIG(4·14) VARIATION OF DEFLECTION, MOMENT, SHEAR FORCE & SOIL PRESSURE WITH LENGTH



FIG(4·15) EFFECT OF VARIATION OF Z_{max} ON LOAD DEFLECTION RELATIONSHIP - CASE 2

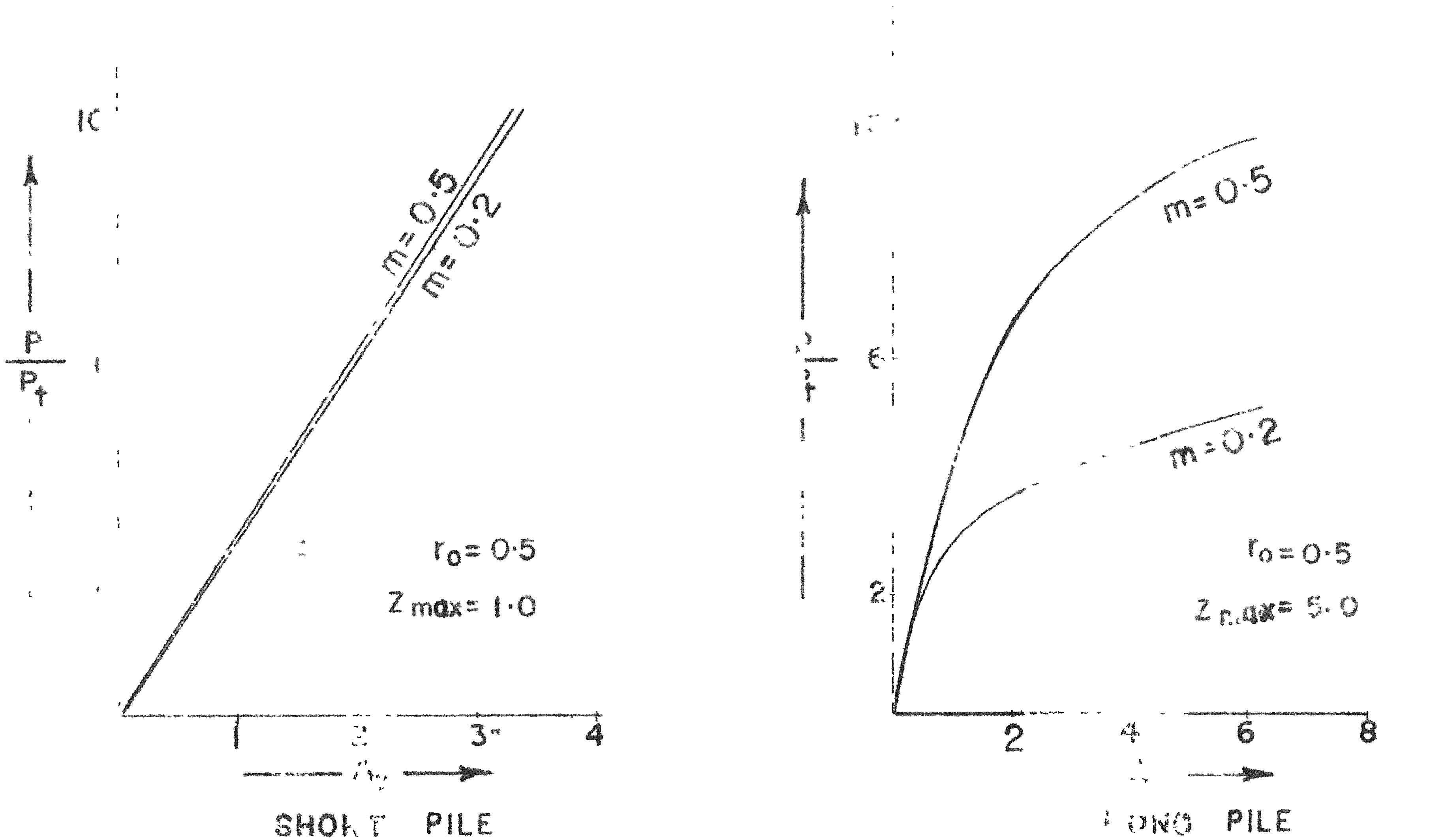
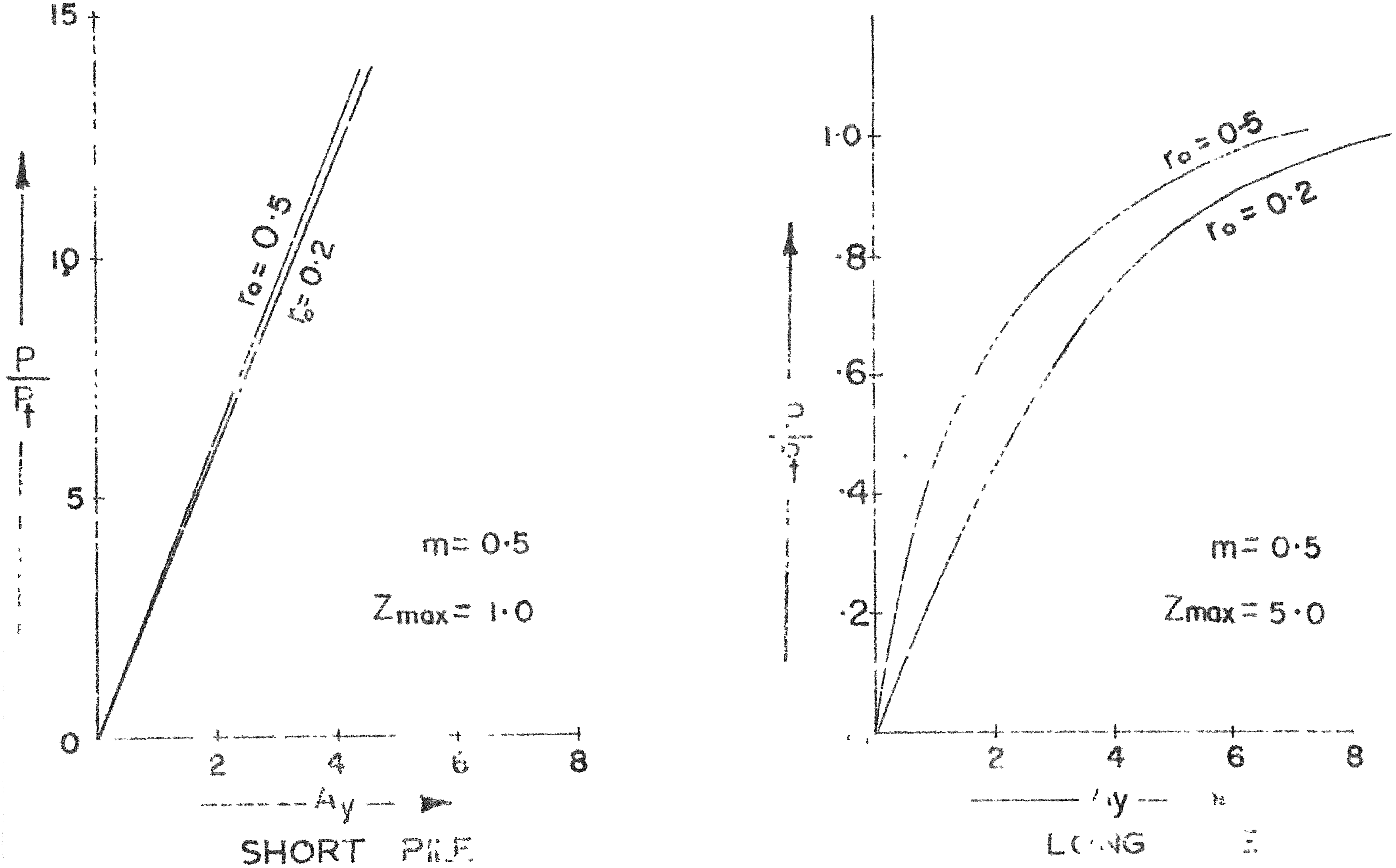


FIG 4·16 EFFECT OF 'm' ON LOAD DEFLECTION RELATIONSHIP

CASE 2



CASE 2

FIG 4.17 EFFECT OF 'r_o' ON LOAD DEFLECTION RELATIONSHIP

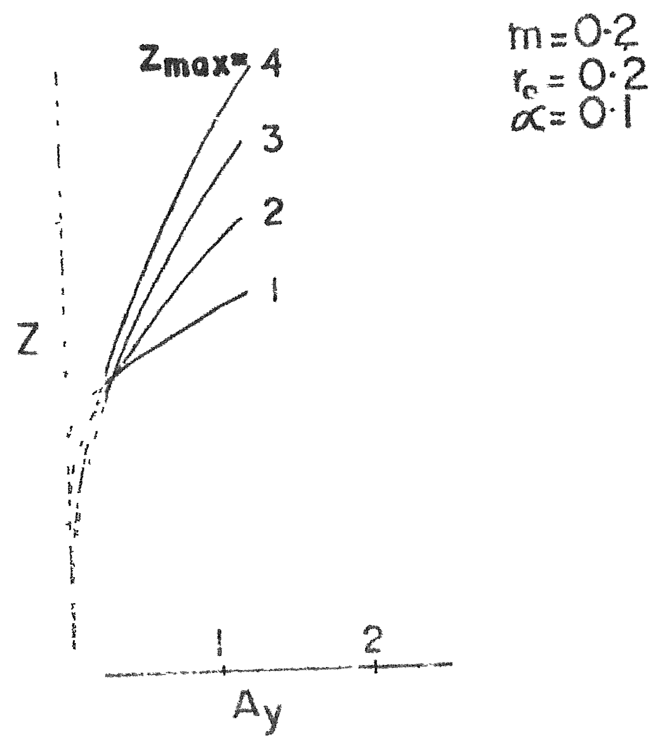
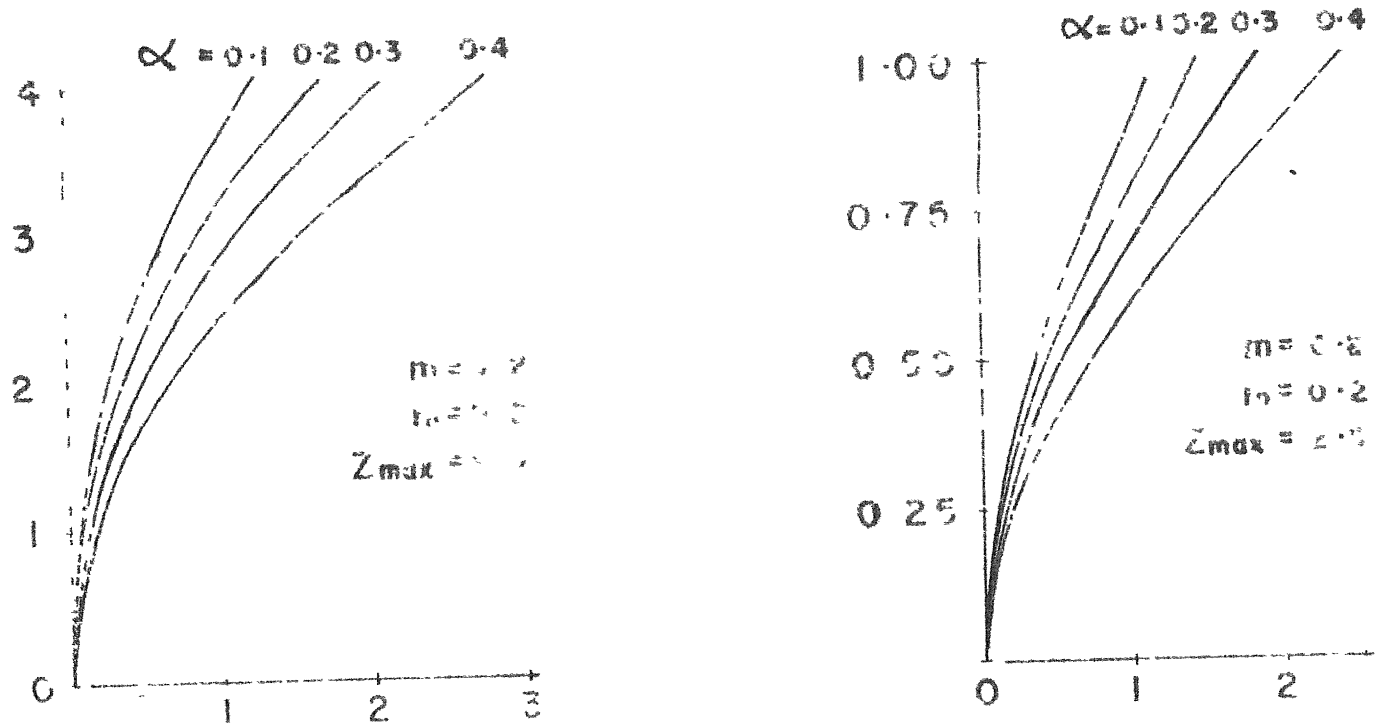


FIG 4·18 EFFECT OF Z_{max} ON DEFLECTION ALONG THE LENGTH

CASE 2

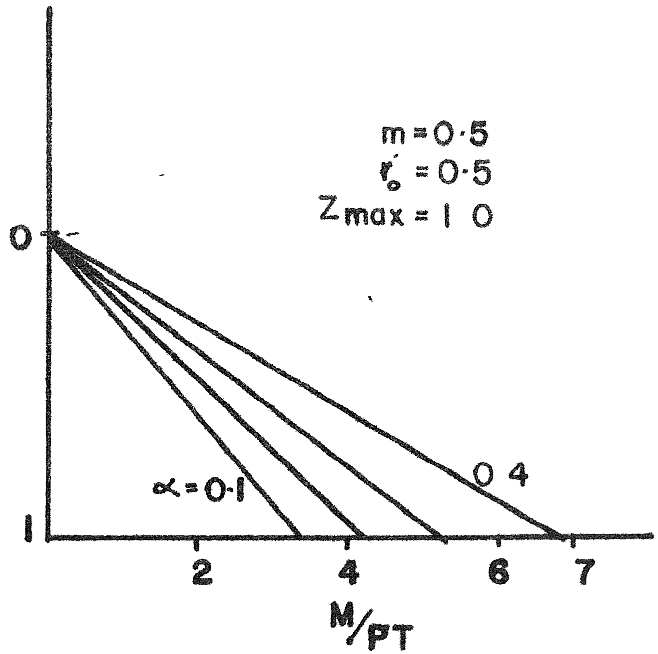


(a) LONG PILE

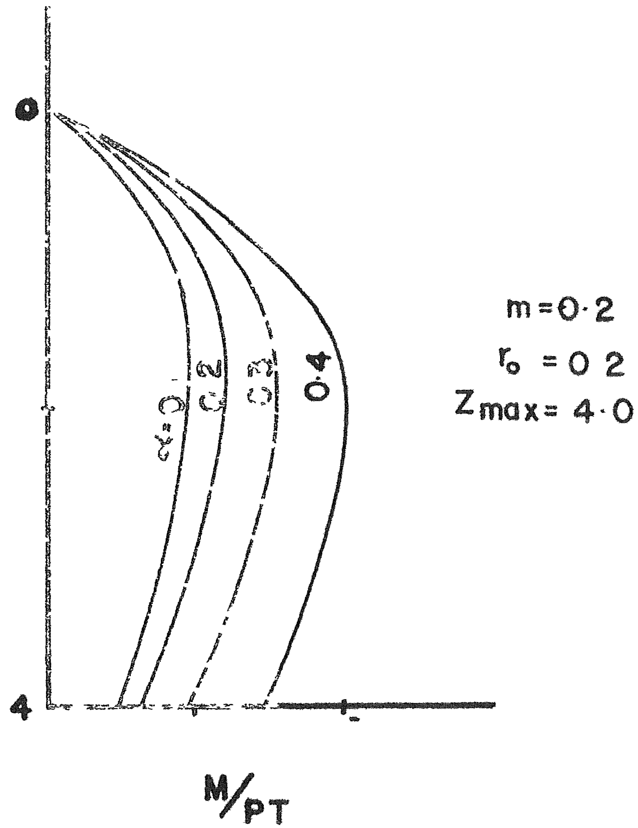
(b) SHORT PILE

FIG (4·19) EFFECT OF 'α' ON DEFLECTION ALONG THE LENGTH

CASE 2



SHORT PILE



LONG PILE

CASE 2

FIG(4·22) EFFECT OF α ON BENDING MOMENT DISTRIBUTION

CHAPTER V

STRESSES IN SOIL DUE TO LATERAL PILE LOADING

5.1 INTRODUCTION

Experimental and theoritical studies (2,3) have been extensively carried out to find the response of a pile to lateral loads acting at the top of the pile. Geddes (44) has derived equations for stresses in the foundation soils due to vertical sub-surface loading. Here an attempt has been made to find the stresses in the soil around the pile when the pile is subjected to a lateral sub-surface loading assuming the soil to be homogeneous, isotropic, elastic medium obeying Hook's law.

5.2 FORMULATION OF THE PROBLEM

Mindlin (45) obtained a set of equations giving the stresses due to a force applied parallel to the boundary below the surface of a semi-infinite medium. These equations are used in solving this problem.

According to Mindlin the vertical stress at a point defined by the rectangular co-ordinates (x, y, z), is given by

$$\sigma(x,z)_{y=o} = \frac{P\,x}{8\pi(1-\mu)}\left\{ \frac{(1-2\mu)}{R_1^3} - \frac{(1-2\mu)}{R_2^3} - 3\,\frac{(z-u)^2}{R_1^5} - \frac{3(3-4\mu)(z+u)^2}{R_2^5} + \frac{6\,u}{R_2^5}\left[u+(1-2\mu)(z+u)+\frac{5\,z\,(z+u)^2}{R_2^2}\right]\right\}$$

. . .(5.1)

where, $R_1^2 = x^2+(z-u)^2$

$R_2^2 = x^2+(z+u)^2$

μ = Poisson's ratio

P = applied load

u = the depth of dP below the surface of medium as given in Fig. (5.1).

The response of a short pile is similar to that of a rigid pile (chapter IV). The deflections of and pressure along the pile vary linearly with depth. Hence a linear load distribution is considered in the present analysis and is shown in Fig. (5.1(b)). It is defined by

$$dP = q\,du = \left[(q_2-q_1)\,u/L+q_1\right] du \quad . . . (5.1.1)$$

where, q_1, q_2=induced stresses in the soil at the top and bottom of the pile.

5.3 SOLUTION:

The stress at any point is obtained by integrating Eqn. (5.1).

$$\sigma(x, z)_{y=o} = \int_0^L \left[\frac{(q_2-q_1)\ u}{L} + q_1\right] \frac{x}{8\pi(1-\mu)}$$

$$\left\{\frac{(1-2\mu)}{R_1^3} - \frac{(1-2\mu)}{R_2^3} - \frac{3(z-u)^2}{R_1^5} - \frac{3(3-4\mu)(z+u)^2}{R_2^5}\right.$$

$$\left. + \frac{6\ u}{R_2^5}\left[u + (1-2\mu)\ (z+u) + \frac{5\ z\ (z+u)^2}{R_2^2}\right]\right\} du$$

$$\ldots\ldots (5.2)$$

Non-dimensionalising, the final expression is

$$\frac{L\sigma}{q_1} = \frac{\beta}{8\pi(1-\mu)}\left[\sum I + (m-1)\sum J\right]$$

where, $\frac{L\sigma}{q_1}$ = the stress factor

$$m = \frac{q_2}{q_1}$$

$$\beta = \frac{x}{L}$$

$$\alpha = \frac{z}{L}$$

$$\sum I = (1-2\mu)(I_1 - I_2) - 3\ I_3 - 3\ (3-4\mu)\ I_4 + (6\ I_5) + 6\ (1-2\mu)\ I_6 + 30\ I_7 \alpha$$

$$\Sigma J = (1-2\mu)\ (J_1 - J_2) - 3\ J_3 - 3\ (3-4\mu) J_4$$

$$+ 6\ J_5 + 6\ (1-2\mu)\ J_6 + 30\ \alpha\ J_7$$

$$I_1 = \frac{1}{\beta^2}\left[\frac{\alpha}{x_3} - \frac{(\alpha - 1)}{x_1}\right]$$

$$I_2 = \frac{1}{\beta^2}\left[\frac{(\alpha + 1)}{x_2} - \frac{\alpha}{x_3}\right]$$

$$I_3 = -\frac{1}{3\beta^2}\left[\frac{(\alpha - 1)^3}{x_1^3} - \frac{\alpha^3}{x_3^3}\right]$$

$$I_4 = \frac{1}{3\beta^2}\left[\frac{(\alpha + 1)^3}{x_2^3} - \frac{\alpha^3}{x_3^3}\right]$$

$$I_5 = \left[1 - \frac{\alpha^2}{\beta^2}\right] I_4 + \frac{\alpha^2}{\beta^2} I_2 + \frac{2\alpha}{3}\left[\frac{1}{x_2^3} - \frac{1}{x_3^3}\right]$$

$$I_6 = I_4 + \frac{\alpha}{3}\left[\frac{1}{x_2^3} - \frac{1}{x_3^3}\right]$$

$$I_7 = I_{71} - \alpha I_{72}$$

$$I_{71} = \frac{1}{3}\left[\frac{1}{x_3^3} - \frac{1}{x_2^3}\right] + \frac{\beta^2}{5}\left[\frac{1}{x_2^5} - \frac{1}{x_3^5}\right]$$

$$I_{72} = \frac{1}{3\beta^2}\left[\frac{(\alpha+1)^3}{x_2^3} - \frac{\alpha^3}{x_3^3}\right]$$

$$- \frac{1}{5\beta^4}\left[\frac{(\alpha+1)^5}{x_2^5} - \frac{\alpha^5}{x_3^5}\right]$$

$$J_1 = \alpha I_1 - \frac{1}{x_1} + \frac{1}{x_3}$$

$$J_2 = -\frac{1}{x_1} + \frac{1}{x_3} - \alpha I_2$$

$$J_3 = \alpha I_3 - \frac{1}{x_1} + \frac{1}{x_3} + \frac{\beta^2}{3}\left[\frac{1}{x_1^3} - \frac{1}{x_3^3}\right]$$

$$J_4 = \frac{1}{x_3} - \frac{1}{x_2} + \frac{\beta^2}{3}\left[\frac{1}{x_2^3} - \frac{1}{x_3^3}\right] - \alpha I_4$$

$$J_5 = J_{51} + J_{52} + J_{53} + J_{54}$$

$$J_{51} = J_4 + \alpha I_4$$

$$J_{52} = -3\alpha I_4$$

$$J_{53} = \alpha^2\left[\frac{1}{x_3^3} - \frac{1}{x_2^3}\right]$$

$$J_{54} = -\frac{\alpha^3}{\beta^2}(I_2 - I_4)$$

$$J_6 = J_{51} - 2\alpha I_4 - \frac{\alpha^2}{3}\left[\frac{1}{x_2^3} - \frac{1}{x_3^3}\right]$$

$$J_7 = J_{71} + J_{72} + J_{73}$$

$$J_{71} = \frac{1}{5\beta^2}\left[\frac{(\alpha+1)^5}{x_2^5} - \frac{\alpha^5}{x_3^5}\right]$$

$$J_{72} = -2\alpha \; I_{71}$$

$$J_{73} = \alpha^2 \quad I_{72}$$

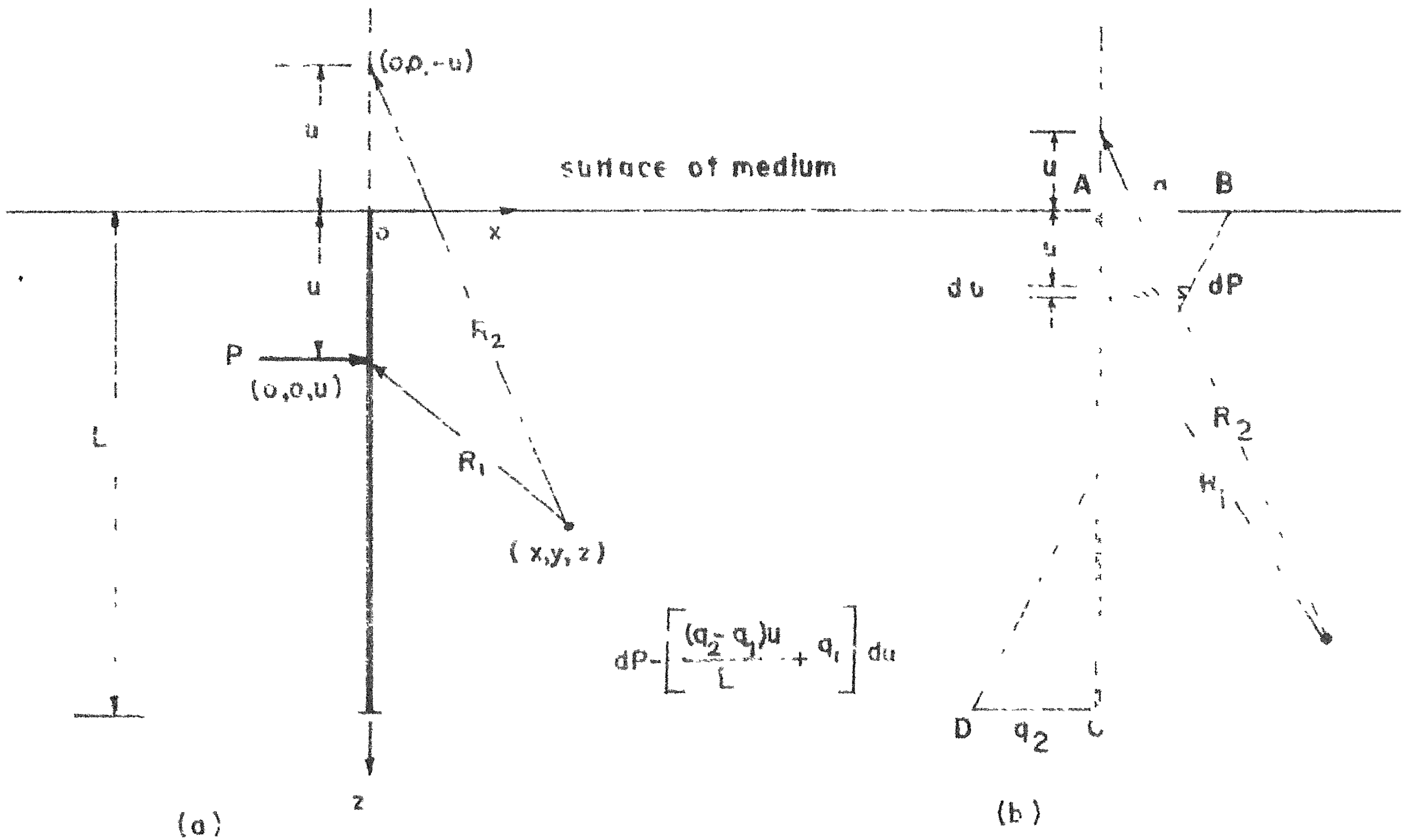
Here the variables are α, β, m and μ. The effect of these variables on the stress factor is studied. The results are shown in the form of graphs as shown in Figs. (5.2), (5.3) and (5.4).

5.4 DISCUSSION OF RESULTS AND CONCLUSION:

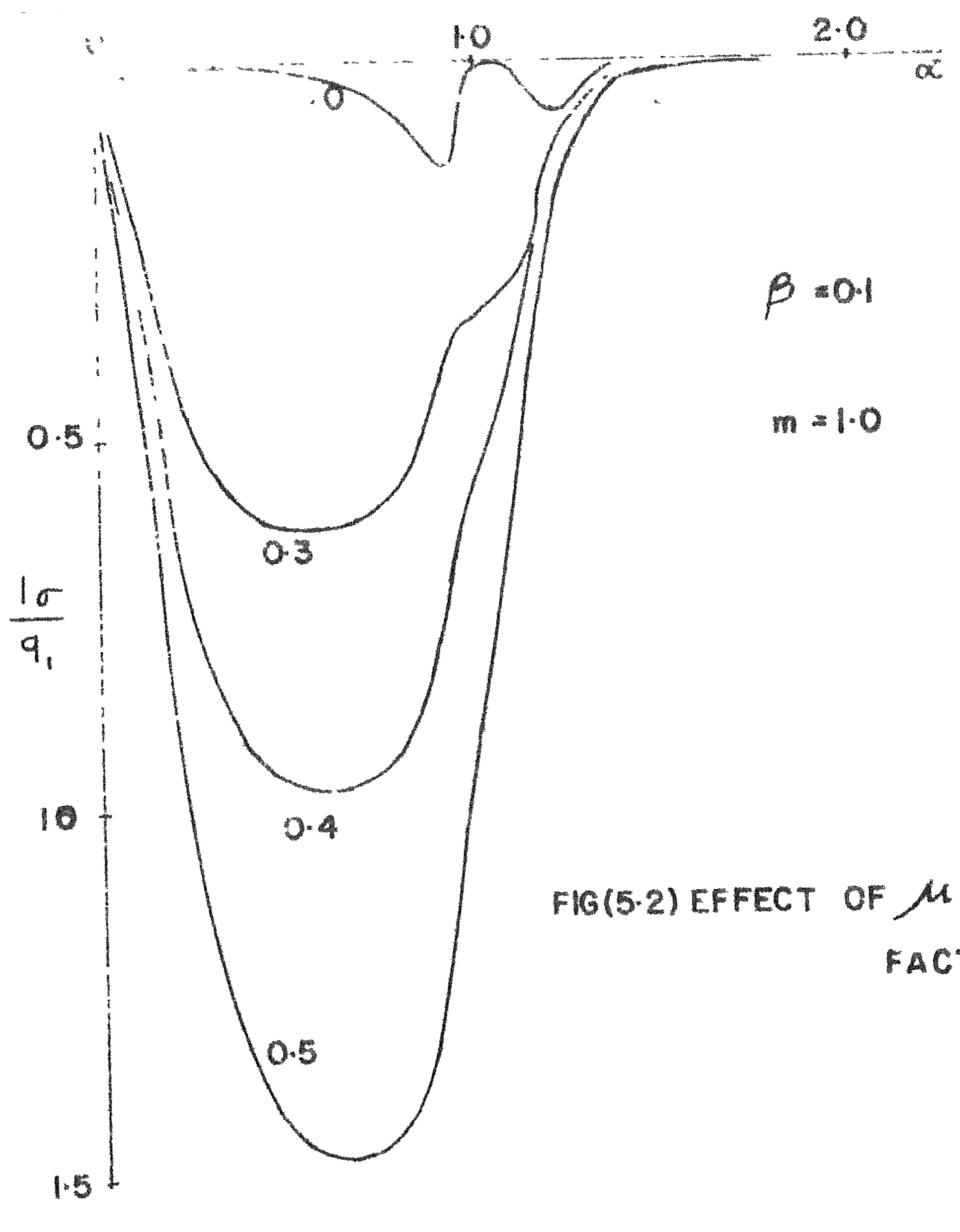
As μ increases the stress factor increases as seen in Fig. (5.2). As β increases the stress factor decreases as seen in Fig. (5.3). As 'm' decreases the stress factor decreases but when 'm' = -2.0, the stress factor decreases to start with and then shoots up as shown in Fig. (5.4). From the figures (5.2), (5.3), (5.4) it is obvious that beyond a depth of L to 2L the vertical stress is not significant.

The pressure bulbs can be drawn for the vertical stresses from the graphs plotted. Aslo the expressions for the stresses in other directions can be derived.

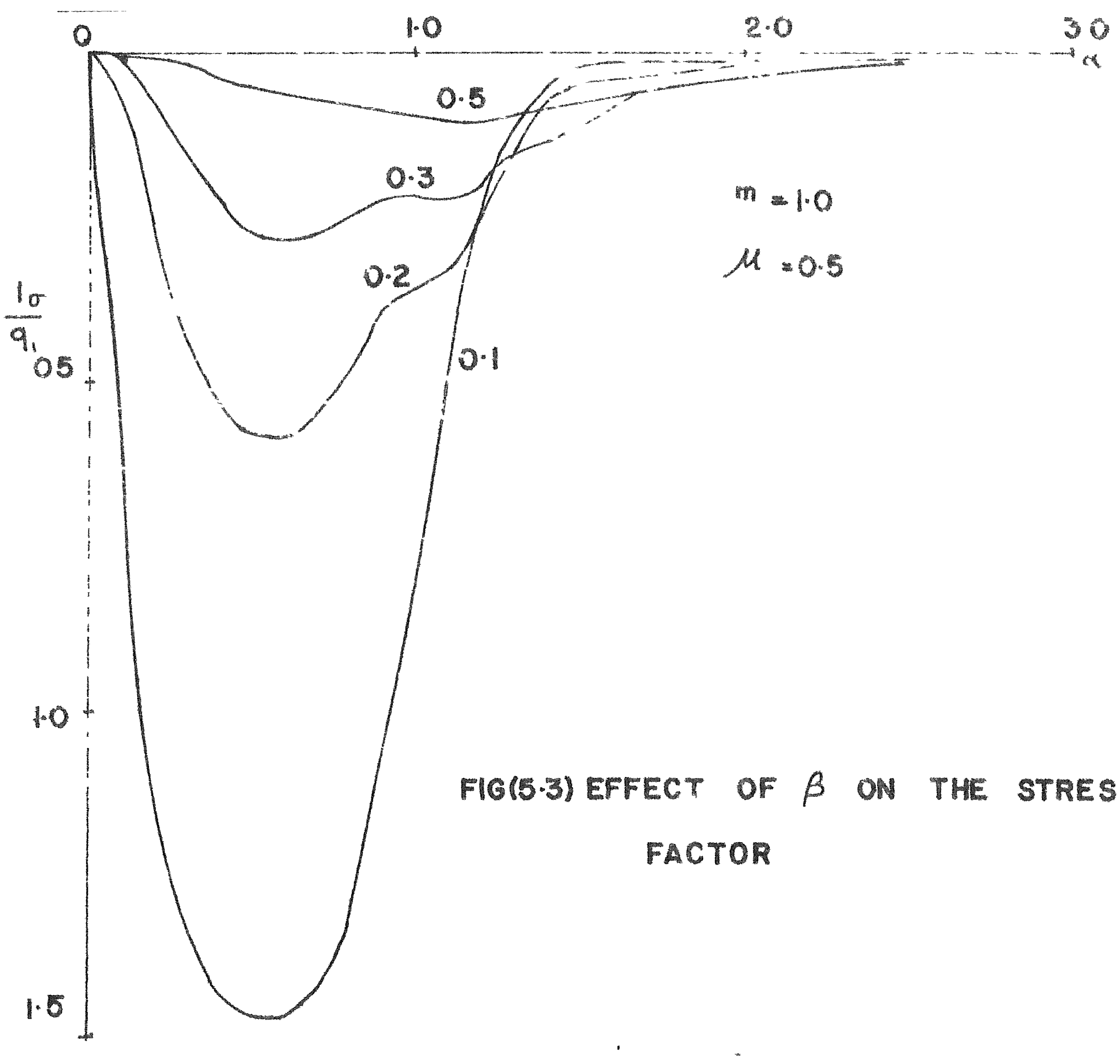
To conclude, knowing the values of α, β, m, μ the vertical stress in the soil due to lateral pile loading can be calculated.



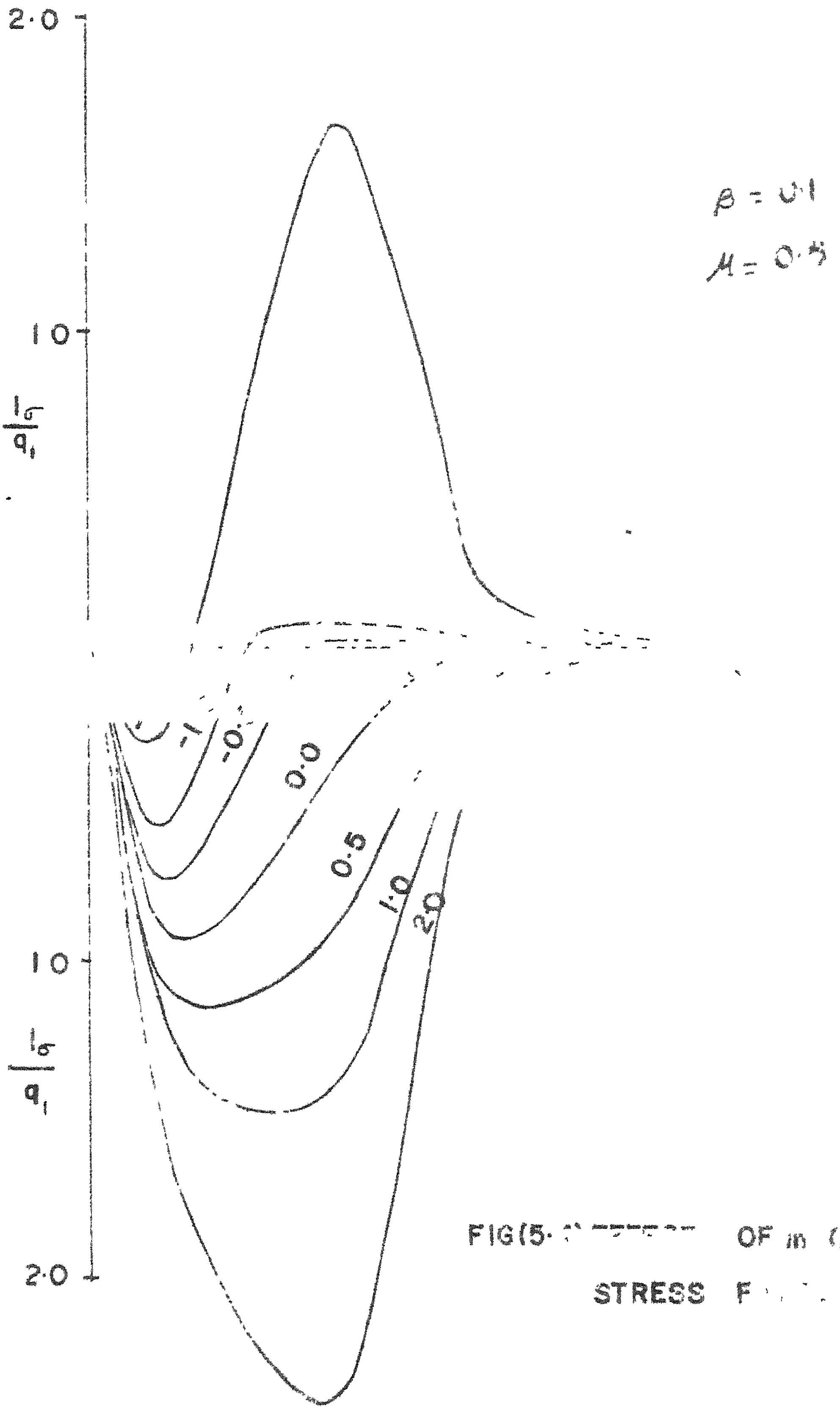
FIG(5.1) LOADING AND STRESS CONDITION



FIG(5·2) EFFECT OF μ ON THE STRESS FACTOR



FIG(5·3) EFFECT OF β ON THE STRESS FACTOR



FIG(5· OF m ON THE

STRESS F

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